1. Let $\{X_n\}_{n\geq 0}$ be a Markov chain on state space S with transition matrix P and initial distribution μ. Let $i \in S, m, n \geq 0$ and

$$Y_m = X_{n+m} \mid X_n = i.$$

Show that Y_m is a Markov chain on state space S with transition matrix P and initial distribution δ_i.

2. Let $\{X_n\}_{n\geq 0}$ be a Markov chain on state space $S = \{1, 2, 3\}$ with transition matrix

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$$

and initial distribution μ.

$$p_{11}^n = \frac{1}{5} + (\frac{1}{2})^n(\frac{4}{5}\cos(\frac{n\pi}{2}) - \frac{2}{5}\sin(\frac{n\pi}{2}))$$

3. Let $0 < p < 1, n \geq 1$. Suppose now that $Z_0, Z_1, \ldots$ are i.i.d Bernoulli (p). Let $S_0 = 0, S_n = Z_1 + Z_2 + \ldots + Z_n$. In each of the following cases determine whether $\{X_n\}_{n\geq 0}$ is a Markov chain:

   (a) $X_n = Z_n$,

   (b) $X_n = S_n$,

   (c) $X_n = S_0 + S_1 + \ldots S_n$.

   In the cases where $\{X_n\}_{n\geq 0}$ is a Markov chain find its state-space and transition matrix. In cases where it is not a Markov chain give an example where $P(X_{n+1} = i \mid X_n = j, X_{n-1} = k)$ is not independent of k.

[1]**Instructions:** Please write up individual solutions for each of the problems and hand it over at 11:00am. Secondly, form 3 groups. Each group should write the solution of one of the problems on the board, Leon, please take the photographs of the solutions and emails it to me.