1. Compute $p_{11}(t)$ for $P(t) = e^{tQ}$ where

$$Q = \begin{pmatrix} -2 & 1 & 1 \\ 4 & -4 & 0 \\ 2 & 1 & -3 \end{pmatrix}$$

2. A random variable $T : \Omega \to (0, \infty]$ has an exponential distribution if and only if

$$P(T > s + t \mid T > s) = P(T > t) \text{ for all } s, t \geq 0$$

3. Suppose S and T are independent exponential random variables of parameters α and β respectively. What is the distribution of $\min\{S, T\}$? Show that events $S < T$ and $\{\min\{S, T\} \geq t\}$ are independent.

4. Let $T_1, T_2, \ldots$ be independent exponential random variables with parameters $\lambda_1, \lambda_2, \ldots$ respectively. Show that $\lambda_1 T_1$ is exponential of parameter 1. Suppose $\sup_n \lambda_n < \infty$ then show that

$$P(\sum_{n=1}^{\infty} T_n = \infty) = 1.$$

5. Let $S_1, S_2 \ldots$ be independent exponential random variables of parameter λ and let N be an independent geometric random variable with

$$P(N = n) = \beta(1 - \beta)^{n-1}$$

for $n = 1, 2, \ldots$ Show that $S = \sum_{i=1}^{N} S_i$ has exponential distribution of parameter $\lambda\beta$.