1. Let X_n be a Markov Chain on $S = \{1, 2\}$ with transition matrix P. Find $\mathbb{P}_1(T^{\{1\}} = n)$ and $\mathbb{P}_1(T^{\{2\}} = n)$

2. Let X_n be the Ehrenfest chain, i.e X_n is a Markov Chain on $\{0, 1, \ldots d\}$, with transition matrix P given by

$$p_{ij} = \begin{matrix} \frac{i}{d} & \text{if } j = i - 1 \\ 1 - \frac{i}{d} & \text{if } j = i + 1 \\ 0 & \text{otherwise} \end{matrix}$$

   (a) Suppose that $X_0 \sim$ Binomial $(d, \frac{1}{2})$, find the distribution of X_1.

   (b) Show that the transition matrix P satisfies,

$$\sum_{i=0}^{d} i p_{ki} = Ak + B,$$

   for all $k \in \{0, 1, 2, \ldots, d\}$, for some constants A, B.

   (c) Show that $\mathbb{E}(X_{n+1}) = A\mathbb{E}(X_n) + B$.

   (d) Find $\mathbb{E}_i(X_n)$ for any $i \in S$.

3. Let $\mathbb{T}^2$ be the rooted binary tree. That is, it is an infinite graph with ρ as the distinguished vertex, which comes with a single edge; at every other vertex there are three edges; and there are no closed loops. Place natural weights μ, on the edges. Show that the random walk on $\mathbb{T}^2, \mu$ is transient.

4. Let $\mathbb{Z}^2$ be the integer lattice. Place natural weights μ, on the edges. Show that the random walk on $\mathbb{Z}^2, \mu$ is recurrent.