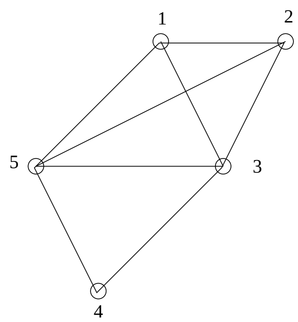
1. Let S be a countable set. Let $P_{|S|\times|S|}$ and $Q_{|S|\times|S|}$ be two stochastic matrices. Then show that the product $R = PQ$ (thus $r_{ij} = \sum_{k\in S} p_{ik}q_{kj},\ \ i,j \in S$) makes sense and is also a stochastic matrix.

2. The examples given below can be modeled by a Markov chain. Determine the state space, initial distribution and the transition matrix for each.

   (a) Suppose N black balls and N white balls are placed in two urns so that each urn contains N balls. At each step one ball is selected at random from each urn and the two balls interchange places. The state of the system at time $n \in \mathbb{N}$ is the number of white balls in the first urn after the n-th interchange.

   (b) Suppose a gambler starts out with a certain initial capital of N rupees and makes a series of 1 rupee bets against the gambling house until her captial runs out. Assume that she has probability p of winning each bet. Let the state of the system at time $n \in \mathbb{N}$ denote her capital at the n-th bet.

   (c) A particle is moving along the graph shown below. At each time step it moves along one of the incident edges to a neighbouring vertex, choosing the edge with equal probability and independently of all previous movements. Assume that it starts at a uniformly chosen point on the graph. Let the state of the system at time $n \in \mathbb{N}$ be the position of the particle at time n.



3. Meteorologist Chakrapani could not predict rainy days very well in the wet city of Cherapunjee. So he decided to use the following prediction model for rain. If it had rained yesterday and today, then it will rain tomorrow with probability 0.5. If it rained today but not yesterday, then it will rain tomorrow with probability 0.3. If it did not rain today but had rained yesterday, then it will rain tomorrow with probability 0.1. Finally if it did not rain today and had not rained yesterday, then it will rain tomorrow with probability 0.1. Let X_n denote R if it rained on day $n \in \mathbb{N}$ and D if it was a dry day (no rain). Assume that with probability 0.5 it rains on day 0.

   Show that $\{X_n : n \geq 0\}$ is not a Markov chain but $(Y_n = (X_n, X_{n-1}), n \geq 1)$ is a Markov chain. Write down the state space, initial distribution and the transition matrix for the chain Y_n.

4. Consider a Markov chain X_n on state space $\{A, B, C\}$ with initial distribution μ and transition matrix given by

$$P = \begin{pmatrix} .2 & .4 & .4 \\ .4 & .4 & .2 \\ .4 & .6 & 0 \end{pmatrix}.$$

   (a) what is the probability of going from state A to state B in one step ?

   (b) what is the probability of going from state B to state C in exactly two steps ?

   (c) what is the probability of going from state C to state A in exactly two steps ?

   (d) what is the probability of going from state C to state A in exactly three steps ?

   (e) Calculate the second, third and fourth power of this matrix. Do you have a guess for P^n for large n