Due : Tuesday, October 14th 2003

Determine the state space and the transition matrix for the following Markov Chains:

1. Consider a population of a particular type of bacteria. At every unit time the bacteria die or give birth to two particles with probability $\frac{1}{2}$. The state of the system at time n is the number of bacteria at time n.

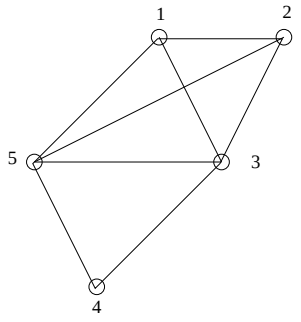
2. N black balls and N white balls are placed in two urns so that each urn contains N balls. At each step one ball is selected at random from each urn and the two balls interchange places. The state of the system at time n is the number of white balls in the first urn after the n-th interchange.

3. Suppose we have two boxes labelled 1 and 2, and d balls labelled $1, \ldots, d$. Initially some of these balls are in box 1 and the remainder is in box 2. A ball is selected at random and placed in the opposite box. This procedure is repeated indefinitely with the selections being independent from trial to trial. Let the state of the system at time denote the number of balls in box 1 after the nth trial.

4. Suppose a gambler starts out with a certain initial capital in rupees and makes a series of 1 rupee bets against the gambling house (i.e if she wins the bet she gets 1 extra rupee and if she looses the bet she looses the rupee). Assume that she has respective probabilities p and $1-p$ of winning and losing each bet. If her capital reaches 0 then it stays there. Let the state of the system at time n denote the capital of the gambler at the n-th bet.

5. Consider a gene composed of d subunits, where d is some positive integer and each subunit is either normal or mutant in form. Consider a cell with a gene composed of m mutant subunits and $d-m$ normal subunits. Before the cell divides into two daughter cells, the gene duplicates. The corresponding gene of one of the daughter cells is composed of d units chosen at random from the $2m$ mutant subunits and the $2(d-m)$ normal subunits. Suppose we follow a fixed line of descent from a given gene. Let the state of the system at time n be number of mutant subunits present in the nth descendant gene.

6. A particle is moving along the graph (shown below). At each time step it moves along one of the adjacent edges to a neighbouring vertex, choosing the edge with equal probability and independently of all previous movements. Let the state of the system at time n be the position of the particle at time n.



Problems to be turned in are: 4,6