Due : Thursday, September 4th 2003

1. Let X be a real valued random variable with a strictly increasing distribution function F. Define $Y = F(X)$. Find the distribution of Y.

2. Let $X_1, X_2, \ldots, X_n$ be i.i.d X, where the density of X is given by $f(x) = \frac{1}{\pi\sigma}\frac{1}{1+(\frac{x-\mu}{\sigma})^2}$. Find the assymptotic distribution of
   (a) Sample median
   (b) $\frac{X_{(\frac{3n}{4})} - X_{(\frac{n}{4})}}{2}$,
   (c) $\frac{X_{(\frac{3n}{4})} + X_{(\frac{n}{4})}}{2}$,

3. Suppose ξ_n and ψ_n are two estimates of a parameter θ. If $\sqrt{n}(\xi_n - \theta) \xrightarrow{d} N(0, \sigma_1^2)$ and $\sqrt{n}(\psi_n - \theta) \xrightarrow{d} N(0, \sigma_2^2)$, then the *Assymptotic efficiency* of ξ_n relative to ψ_n is defined to be the ratio $\frac{\sigma_2^2}{\sigma_1^2}$.
   (a) Let $X_1, X_2, \ldots, X_n$ be i.i.d. $N(\mu\sigma^2)$. Find the Assymptotic efficiency of $X_{([\frac{n}{2}])}$ relative to the sample mean.
   (b) Discuss the meaning of Assymptotic efficiency.

4. (Ferguson, page 93, Problem 6.) Let $X_1, \ldots, X_n$ be a sample from the beta distribution with density, $f(x) = \theta x^{\theta-1} 1(0 < x < 1)$, where $\theta > 0$.
   (a) Describe $\xi_{\frac{1}{2}}$ as a function of θ.
   (b) Let $\hat{\theta}_n = \frac{-\log 2}{\log(X_{(\frac{n}{2})})}$. Does $\hat{\theta}_n$ converge in probability ?
   (c) Find the assymptotic distribution of $\hat{\theta}_n$

Problems to be turned in are : 2(b), 3(a), (4)