

Question: Let $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ be sequences of real numbers and let $\{z_n\}_{n=1}^{\infty}$ be a sequence of real numbers such that $z_n = x_n + y_n$ for all $n \in \mathbb{N}$. Show that

$$\liminf_{n \rightarrow \infty} z_n \geq \liminf_{n \rightarrow \infty} x_n + \liminf_{n \rightarrow \infty} y_n \quad (1)$$

when¹ $-\infty < \liminf_{n \rightarrow \infty} x_n < \infty$ and $-\infty < \liminf_{n \rightarrow \infty} y_n < \infty$.

¹the inequality (1) is true provided that the sum on the right hand side is not of the form $\infty - \infty$.