**Due: April, 10th 2003.**

Let $(\Omega, \mathcal{F}, P)$ be a probability space. Let $B_t$ be a Brownian motion adapated to the filtration $\mathcal{F}_t$

1. Let the market rate be given by an adapted process $r(t)$. Suppose the stock price process $S(t)$ satisfies the following Stochastic Differential equation

$$dS(t) = \mu(t)S(t)dt + \sigma(t)S(t)dB(t),$$

with $\sigma$ and $\mu$ being adapted to $\mathcal{F}_t$.

   (a) Describe the risk neutral measure via the radon nikodym derivative with respect to $P$.

   (b) Using the martingale representation theorem, describe the hedging portfolio process $\Delta(\cdot)$ in terms of $\sigma$, $\mu$, $r$ and $S$ for a contigency claim with payoff $V$ at time $T$.

2. Let $\sigma(t) \equiv \sigma$, $\mu(t) \equiv \mu$, $r(t) \equiv r$. (i.e. all these functions are constant in time). Suppose there are two stock prices $S$ and $S1$ where $S1(t)$ satisfies the following Stochastic Differential equation

$$dS1(t) = \mu 1 S1(t)dt + \sigma 1 S1(t)dB(t).$$

If $\frac{\mu - r}{\sigma} \neq \frac{\mu 1 - r}{\sigma 1}$ then show that the market admits arbitrage.