Due: February 14th.

Consider the Binomial model with $n = 200$ arbitrary r, u, d. Let $S_0 = 10$.

1. Let X be a random variable on the probability space $(\Omega, \mathcal{F}, P)$. If g is a convex function, then it is known that $E(g(X) \mid \mathcal{G}) \geq g(E(X \mid \mathcal{G}))$. Consider an American call with payoff $g(S_k)$ if exercised at time k. Consider an European call with pay off $g(S_{200})$. Then show that the value at time zero of both these calls is the same.

2. Let $\tau = 3$ for all $\omega \in \Omega$. Show that τ is a stopping time.

3. Let $\eta = \min\{k : S_k = 1\}$. Show that η is a stopping time.

4. If S and T are stopping times then show that $S + T$, $\min(S, T)$ and $\max(S, T)$ are stopping times.

5. Suppose that Z_k is a martingale with respect to $\mathcal{F}_k$. Let τ be a stopping time. Show that $Z_{\min(\tau,k)}$ is a martingale with respect to $\mathcal{F}_k$.

6. Suppose $p = \frac{1}{200}$ and $u = 2$, what would the values of d have to be so that S_k would be a martingale under the market probability with respect to $\mathcal{F}_k$.

7. Let Y_0, p and q be arbitrary numbers. Let $\{\xi_i : i \in N\}$ be independent and identically distributed random variables such that $\xi_i(H) = 1$ and $\xi_i(T) = -1$. Let $Y_k = \sum_{i=1}^{k} \xi_i + Y_0$, for $k \geq 1$.

   (a) Investigate when is Y_k a martingale, sub-martingale and a super-martingale.

   (b) Assume $p = 1 = q = \frac{1}{2}$. Show that $Y_k^2 - (k + 1)$ is a martingale.

   (c) Let $Y_0 = 4$. Suppose $\tau = \min\{k : Y_k = 2 \text{ or } Y_k = 5\}$. Find the $P(Y_\tau = 5)$