Due: February 7th.

Key words-
Mathematics: Stopping times
Finance:- American option, Put

1. Define the following options: American put.

2. Define the follwing positions: Short, Long.

3. Let $S_0 = 4$, $u = 2$, $d = \frac{1}{2}$, $r = \frac{1}{4}$. Let $n = 2$. Consider the binomial model with the above parameters and an american call with strike price 5. Let $v_k(x)$ be the value of the option at time k if the stock price is x. Define v_k as was done in class.

   (a) Calculate v_k for each k.

   (b) Assume that the value of the option is v_k. To Hedge this option assume(incorrectly) that the self-financing wealth process was the same as the one for European. $\Delta_1(T)$ will appear in two equations. Find the values from both and conclude that they are not the same.

   (c) Explain the reasoning for the above.

4. Do Example 5.2, 5.3 and 5.4 with $n = 2$, $S_0 = 5$, $u = 2$ $d = \frac{1}{2}$, $r = \frac{1}{4}$ and Strike price 6.

5. Define what is meant by an American derivative security. (page 85). Let G_k be an American derivative security. Let $V_k = \max_\tau (1+r)^k \tilde{E}[(1+r)^{-\tau} G_\tau \mid \mathcal{F}_k]$, where the maximum is over all stopping times τ satisfying $\tau \geq k$.

   (a) Show that $V_k \geq G_k$

   (b) What is a super-Martingale ?. Show that $(1+r)^{-k} V_k$ is a super-martingale.

   (c) Let $C_k = V_k - \frac{1}{1+r}\tilde{E}[V_{k+1} \mid \mathcal{F}_k]$. Let

   $$\Delta_k(\omega_1, \dots, \omega_k) = \frac{V_{k+1}(\omega_1, \dots, \omega_k, H) - V_{k+1}(\omega_1, \dots, \omega_k, T)}{S_{k+1}(\omega_1, \dots, \omega_k, H) - S_{k+1}(\omega_1, \dots, \omega_k, T)}.$$

   Set $X_0 = V_0$ and define recursively

   $$X_k = \Delta_k S_{k+1} + (1+r)(X_k - C_k - \Delta_k S_k).$$

   Then $X_k = V_k$ for all k.