

Due: January 31st.

Key words-

Mathematics: independence, Markov property.

Finance:- Lookback option, Knockout Barrier option, American option.

1. Consider the binomial model with $n = 4$. (a) For a random variable X , Let $\sigma(X)$ denote the sigma algebra generated by X (see Section 1.2). Find $\sigma(S_3)$. Is $\sigma(S_3)$ independent of \mathcal{F}_2 . (b) Let \mathcal{H} denote the set of outcomes determined by the third toss. Show that this is independent of \mathcal{F}_2 .
2. Let $E(X | Y)$ be defined as $E(X | \sigma(Y))$. Consider the binomial model with for a generic p, q, u, d . Using the independence lemma discussed in class. Find $E(S_{k+1} | S_k)$ for $k < n$.
3. Using the Markov property now compute $E(S_3 | \mathcal{F}_2)$.
4. Consider a European Call with expiration time 5 and strike price 10. Assume the binomial model for the stock price with $p = 0.99, u = 2, d = \frac{1}{2}, r = \frac{1}{4}, S_0 = 4$. Define $v_k(x)$ to be the value of the call at time k when $S_k = x$. Describe $v_3(x)$ as a function of x .
5. Do the above problem with $p = 0.5$ and $p = 0.4$.