Ground Rules:

1. **You may work in groups of (atmost) two and can submit one solution for each group.** *All the individuals in a group will be equally responsible for the solution.*
2. *Please return the answer script (print out if needed) to Prof. Muthuramalingam by 1.10pm on February 3rd, 2005.*
3. *It is suggested that you spend the first 10 minutes in class reading the entire worksheet and thinking about how to solve the questions given. After which you may move to the demolab or CC and (are strongly encouraged to) work together on this and help each other out.*
4. *The problems in this worksheet are designed so that you will have just enough time to do the work so please do not waste time.*

Taylor Series: Truncation error

Please read the introductory paragraph below and try to answer the following questions as best as you can.

Fix an $x_0 \in [a,b]$. Let f be a "sufficiently differentiable" function on the interval $[a,b]$. Then by Taylor's Theorem we have:

$$f(x) = f(x_0) + \sum_{k=1}^{n} \frac{(x-x_0)^k}{k!}\frac{d^k f}{dx^k}(x) + \frac{(x-x_0)^{n+1}}{n+1!}\frac{d^{n+1} f}{dx^{n+1}}(\xi),$$

where ξ is a point between x and x_0. The last term in the above expansion is usually referred to as the remainder or the *Truncation error*. One usually writes the above in compact form as

$$f(x) = P_n(x) + O((x-x_0)^{n+1}),$$

where $P_n(x) = f(x_0) + \sum_{k=1}^{n} \frac{(x-x_0)^k}{k!}\frac{d^k f}{dx^k}(x)$. P_n(called Taylor polynomials) is said to be an nth order approximation to f or one says that P_n provides an approximation to f with truncation error of the order of $O((x-x_0)^{n+1})$. Such order of magnitude approximations are an important complement to precise numerical values.

Solve the following questions:

1. Consider the function $f(x) = \frac{1}{1-x}$. Let $a = 1.2$ and $b = 2$. Now write a function file called `Taylor` that will take as input x_0 and dx. The function should do the following:
   (a) Calculate the Taylor polynomials P_1, P_2 and P_3.
   (b) Plot these along with f in the same graph in the interval $(x_0 - \frac{dx}{2}, x_0 + \frac{dx}{2})$ with 20 points.
2. Archimedes estimated π by computing the perimeter of a sequence of polygons inscribed inside a circle of known diameter d. Let a be the length of each side in a n-sided polygon. Then one can show that the perimeter p satisfies

$$p = nd \sin \frac{\pi}{n}$$

The approximation now was that $d * \pi$ the circumference of the circle was close to p for large n. Let $E(d, n) = d(n \sin \frac{\pi}{n} - \pi)$. Show that $E(d, n) = O(n^{-2})$.