

Bonus Questions¹

Without quoting theorems and results please solve the following.

1. Let $(x_i, f(x_i), f'(x_i)), i = 1, \dots, n$ be given. Let

$$P_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3,$$

be the cubic interpolant in the range $[x_i, x_{i+1}]$. Show that under the following constraints:

$$P_i(x_i) = f(x_i), P'_i(x_i) = f'(x_i), P_i(x_{i+1}) = f(x_{i+1}), P'_i(x_{i+1}) = f'(x_{i+1}), 1 \leq i \leq n-1,$$

$$\begin{aligned} a_i &= f(x_i), \\ b_i &= f'(x_i), \\ c_i &= \frac{3f[x_i, x_{i+1}] - 2f'(x_i) - f'(x_{i+1})}{(x_{i+1} - x_i)} \\ d_i &= \frac{f'(x_i) - 2f[x_i, x_{i+1}] + f'(x_{i+1})}{(x_{i+1} - x_i)^2} \end{aligned}$$

2. Complete the proof of cubic splines outlined in class for all the three possibilities.

¹The (best) complete answer will get half a kilogram of mangoes and a loaf of Vanilla icecream.