Due: Thursday February 24th 2005
Problems to be turned: 1,2

1. Consider the 2×2 non-linear system given by

$$
\begin{aligned}
x_1 - x_2 - 3 &= 0 \\
x_1^2 - 20x_1 - x_2 + 5 &= 0
\end{aligned}
$$

   (a) Identify $A(x), b(x)$ such that the above can be written as $A(x)x = b(x)$. Modify `demoSSub` to obtain a solution of the same. Use 10 iterations and find two initializing vectors that give you the two solutions.

   (b) Identify f such that the above can be written as $f(x) = 0$. Modify `demoNewtonSys` to obtain a solution of the same. Use 10 iterations and find two initializing vectors that give you the two solutions.

2. Consider the matrices

$$
W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10^{-6} \end{bmatrix}, A = \begin{bmatrix} 0.1 & 0.1 & 10^6 \\ 0.2 & -0.1 & 10^6 \\ 0.1 & 0.2 & 0 \end{bmatrix}, b = \begin{bmatrix} 0.2 + 10^6 \\ 0.1 + 10^6 \\ 0.3 \end{bmatrix}
$$

   (a) Using MATLAB and the ∞ norm, compute (exactly) $\kappa(A)$. Decide whether the matrix is ill-conditioned or not.

   (b) Using the \ operator in MATLAB, solve $Ax = b$.

   (c) Perturb a_{13} to get another matrix $A + \delta A$ such that $\frac{\|\delta A\|}{\|A\|} \sim 10^{-6}$. Using the \ operator in MATLAB, solve $(A + \delta A)\hat{x} = b$.

   (d) Computer $\frac{\|x - \hat{x}\|}{\|x\|}$

   (e) Let $\tilde{A} = AW$. Do parts (a) -(d) for $\tilde{A}$. Conclude that this matrix A was an "artificially" ill-conditioned matrix.