Due: Thursday February 17th, 2005.
Problems to be turned in 4,6,7

1. Consider the following three vectors:

$$\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}, \text{ and } \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$

   (a) Verify by hand if the vectors are linearly independent or not.
   (b) Use the `rank` function in MATLAB to do the same for above three vectors and the following:

$$\begin{bmatrix} 1 \\ 2 \\ -5 \\ -4 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 7 \\ -6 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 3 \\ 8 \end{bmatrix} \text{ and } \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

   (c) Write a function `length` that will produce as output the $\|\cdot\|_2$ of the vectors in $\mathbb{R}^n$. Test the same with the above vectors.

2. In each part of this problem, you are given the **augmented matrix** of a linear system of equations. For each, describe whether the system is consistent or not. If consistent describe its solution set by indicating clearly the free variables. If inconsistent explain clearly why it is so.

$$\begin{pmatrix} 1 & 0 & 1 & 2 & 1 & 4 \\ 0 & 3 & 3 & 0 & -3 & 0 \\ 0 & 0 & 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 & 4 & 0 \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 & 1 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & -2 \end{pmatrix}$$

3. The **augmented matrix** of a homogeneous system $Qx = 0$ is given below:

$$\begin{pmatrix} 1 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 2 & 2 & 2 & -6 & 0 \end{pmatrix}$$

   (a) Describe the solution set of the homogeneous system $Qx = 0$ in parametric form.
   (b) Suppose $\{x : Qx = 0\} = \text{Span}\{v_1, v_2, \ldots v_p\}$ identify p and then the vectors $v_1, v_2, \ldots v_p$.

4. Solve the following questions **giving reasons** for your answer.

   (a) Suppose $A^{-1} = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$. Without solving for A, find $x_{3\times 1}$ such that

$$x^T A = [1 \quad 0 \quad 0].$$

   (b) Let $B_{34\times 34}$ be a non-singular matrix. Let $A_{34\times 34}$ be a singular matrix. Is AB singular or non-singular ?

(c) Let

$$\left\{ v_1 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \quad , \quad v_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \right\}$$

Is there a vector v_3 different from v_1, v_2 so that set $\{v_1, v_2, v_3\}$ is a **linearly dependent** set of vectors? Justify.

5. Let a_1, a_2, a_3 be the columns of the matrix $A_{4\times3}$ and $b_{4\times1} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$. Provided for you are the results of row reducing the **augmented matrix** $(A|b)$ to its reduced echelon form:

$$(A|b) \rightarrow \begin{pmatrix} 1 & 0 & 0 & -0b_2 + b_1 \\ 0 & 1 & 0 & b_2 - 2b_1 \\ 0 & 0 & 1 & -3b_3 + b_1 + 7b_2 \\ 0 & 0 & 0 & 5b_4 - 19b_2 + 2b_3 - b_1 \end{pmatrix}$$

(a) What condition does $b_{4\times1}$ need to satisfy such that $b_{4\times1}$ is in the $\text{Span}\{a_1, a_2, a_3\}$?

(b) Decide whether $v_{4\times1} = \begin{bmatrix} 5 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ and $w_{4\times1} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$ are in $\text{Span}\{a_1, a_2, a_3\}$. Justify.

6. Write a `parabola` function to automatically set up and solve the system of equationsfor a parabola defined by $y = c_1x^2 + c_2x + c_3$. The function definition should be
`function c = parabola(x,y)`

The function should take two input vectors x and y, each of length three, that define three points through which the parabola passes. The function should return

(a) a vector c of the three coefficients.

(b) a plot of the parabola with the input points shown on the graph.

Test your answer with the following points:

(a) (-2,-1), (0,1), (2,2)

(b) (-2,-2), (-1,-2), (-1,2)

7. The following system of equations, $A_{2\times2}x_{2\times1} = b_{2\times1}$ is singular

$$\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \end{bmatrix}$$

Find the four solutions using "\" operator, obtained when the elements of A and b are perturbed as follows:($\delta = 5 \times 10^{-9}$)

(a) $a_{21} = 2 + \delta$

(b) $a_{22} = 1 + \delta$

(c) $a_{11} = 2 + \delta$, $b_1 = 6 + \delta$

(d) $a_{21} = 2 + \delta$, $b_2 = 6 + \delta$

Does the operator return the correct result when $\delta = 100*$`realmin` ?