Due: Wednesday, October 3rd, 2012
Problem to be turned in : 2,4

1. Let $\{x_n\}_{n=1}^{\infty}$be a sequence of real numbers. Let $S = \limsup_{n\to\infty} x_n$ and $I = \liminf_{n\to\infty} x_n$.
   (a) Prove or disprove: $\mid S \mid = \limsup_{n\to\infty} \mid x_n \mid$ and $\mid I \mid = \liminf_{n\to\infty} \mid x_n \mid$.
   (b) Suppose $S = I = x$ then show that $x_n \to x$ as $n \to \infty$.
2. Let $\{x_n\}_{n=1}^{\infty}$and $\{y_n\}_{n=1}^{\infty}$be sequences of real numbers and let $\{z_n\}_{n=1}^{\infty}$be a sequence of real numbers such that $z_n = x_n + y_n$ for all $n \in \mathbb{N}$. Show that
   (a) $\liminf_{n\to\infty} z_n \geq \liminf_{n\to\infty} x_n + \liminf_{n\to\infty} y_n$
   provided the sum on the right hand side is not of the form $\infty - \infty$.
   (b) $\limsup_{n\to\infty} z_n \leq \limsup_{n\to\infty} x_n + \limsup_{n\to\infty} y_n$
   provided the sum on the right hand side is not of the form $\infty - \infty$.
3. Suppose $\{x_n\}_{n=1}^{\infty}$is a sequence of real numbers. Let $s_n = \sup\{x_k : k \geq n\}$ and $t_n = \inf\{x_k : k \geq n\}$.
   (a) Show that $s_n \to s$ for some $s \in \bar{\mathbb{R}}$ and further that s is a limit point of the sequence $\{x_n\}_{n=1}^{\infty}$.
   (b) Show that $t_n \to t$ for some $t \in \bar{\mathbb{R}}$ and $t = \liminf_{n\to\infty} x_n$.
4. Let $\{x_n\}_{n=1}^{\infty}$be a sequence of real numbers. Suppose the sequence is not bounded. Then show that there is either a subsequence $\{x_{n_k}\}_{k=1}^{\infty}$such that $x_{n_k} \to \infty$ or $x_{n_k} \to -\infty$.
5. Let $\{x_n\}_{n=1}^{\infty}$be a sequence of positive real numbers. Then show that

$$\liminf_{n\to\infty} \frac{z_{n+1}}{z_n} \leq \liminf_{n\to\infty} \sqrt[n]{z_n} \leq \limsup_{n\to\infty} \sqrt[n]{z_n} \leq \limsup_{n\to\infty} \frac{z_{n+1}}{z_n}.$$