Due: Wednesday, August 29th, 2012
Problems to be turned in: 2,4,8

1. If $x > -1$ then $(1+x)^n \geq 1 + nx$ for all $n \in \mathbb{N}$.
2. Let $x \in \mathbb{R}$, $C > 0$, and $\{x_n\}_{n=1}^{\infty}$ be a sequence of real numbers. Show that the following are equivalent:
   (a) $\forall \epsilon > 0$, there is an $N \equiv N_\epsilon \in \mathbb{N}$ such that $\mid x_n - x \mid < \epsilon$ for all $n \geq N$.
   (b) $\forall \epsilon > 0$, there is an $M \equiv M_\epsilon \in \mathbb{N}$ such that $\mid x_n - x \mid \leq C\epsilon$ for all $n > M$.
3. Let $\{x_n\}_{n=1}^{\infty}$ be a sequence of real numbers $\mathbb{R}$ and suppose that $x_n \to x$.
   (a) Let $m, n \in \mathbb{N}$ and $y_n = x_{m+n}$ for all $n \geq 1$. Show that $y_n \to x$ as $m \to \infty$.
   (b) Show that $\{\mid x_n \mid\}_{n=1}^{\infty}$ also converges
4. Find: (i) $\lim_{n\to\infty} \frac{2^n}{n!}$, (ii) $\lim_{n\to\infty} \sqrt{n^2 - n} - n$ and, (iii) $\lim_{n\to\infty} nb^n$, where $b \in (0, 1)$.
5. Consider the $\{y_n\}_{n=1}^{\infty}$, such that $y_1 > 1$ and $y_{n+1} := 2 - \frac{1}{y_n}$ for $n \geq 2$. Show that y_n converges.
6. Suppose $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ are such that for every $\epsilon > 0$ there is an M such that $\mid x_n - y_n \mid < \epsilon$ for all $n \geq M$. If $x_n \to x$ then does it imply that y_n converges.
7. Let A be a bounded non-empty subset of $\mathbb{R}$. Let $s = \sup(A)$ and $i = \inf(A)$. Show that there are sequences $\{x_n\}_{n=1}^{\infty}$, $\{z_n\}_{n=1}^{\infty}$ in A such that $x_n \to s$ and $z_n \to i$ as $n \to \infty$.
8. Give Examples of the following:
   (a) a bounded sequence $\{z_n\}_{n=1}^{\infty}$ that does not converge.
   (b) sequences $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ that do not converge but their sum converges
   (c) sequences $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ that do not converge but their product converges.