Due: Tuesday, October 17th, 2006

Problems to be turned in:1,3,6

1. Assume $a_n > 0$ and that $\sum_{n=1}^{\infty} a_n < \infty$. Does it imply that $\sum_{n=1}^{\infty} \sqrt{a_n a_{n+1}} < \infty$?

2. Find the radius of convergence of the series $\sum_{n=1}^{\infty} nx^n$ and with the radius of convergence give an explicit expression for its sum.

3. Let a_n be a sequence of real numbers and a_{n_k} be a subsequence of the same. Suppose $\sum_{n=1}^{\infty} a_n$ converges. Does it imply that $\sum_{k=1}^{\infty} a_{n_k}$ converge ?

4. Let a_n be a sequence of non-negative real numbers and suppose that $\lim_{n\to\infty} a_n = 0$ and $\sum a_n = \infty$. Show that for every $0 < p < q$, there is a finite set of terms $\{a_{n_1}, \ldots, a_{n_k}\}$ with $p < \sum_{i=0}^{k} a_{n_i} < q$.

5. We say that a series diverges unconditionally to ∞ if every rearrangement of the terms diverges to ∞. Show that a series $\sum_{i=0}^{\infty} x_i$ diverges unconditionally to ∞ if and only if the sum of positive terms of $\{x_i\}$ diverges to ∞ and the sum of the negative terms of $\{x_i\}$ converges.

6. Let $p \in \mathbb{R}$. Decide whether $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{(n+1)^p}$ converges or not.

7. (*Extra credit*) Suppose two teams play a series of games, each producing a winner and a loser, until one team has won two more games than the other. Let G be the total number of games played. Assuming each team has chance 0.5 to win each game, independent of results of the previous games. Find the expected value of G.