Due: Tuesday, August 1st, 2006

Problems to be turned in: 5,7.

In this assignment: $\mathbb{N}$ will denote natural numbers, $\mathbb{Q}$ will denote rational numbers,$\mathbb{R}$ will denote real numbers and $\mathbb{C}$ will denote complex numbers.

1. Show that if $0 < a < b \in \mathbb{R}$, $n \in \mathbb{N}$ then
$$b^n - a^n < (b-a)nb^{n-1}.$$
2. Let $n \in \mathbb{N}$. Prove that $n^3 + 2n$ is always a multiple of 3.
3. For $x > 0, n \in \mathbb{N}$, let $A = \{t \in \mathbb{R} : t > 0, t^n < x\}$. Let $y = \sup(A)$. Show that $y^n > x$ is not possible.
4. (*Rudin: page 21*)Let r and x be real numbers. If r is rational ($r \neq 0$)and x is irrational, prove that $r + x$ and rx are irrational
5. (*Rudin: page 22*) Let A be a non-empty set of real numbers which is bounded below. Let $-A := \{x \in \mathbb{R} : -x \in A\}$. Show that $\inf A = -\sup(-A)$.
6. (*From Rudin: page 22*) Show that the set of all complex numbers $\mathbb{C}$ defined in class is a Field. Decide whether this field can be: (a) ordered set and/or (b) ordered field.
7. (*Rudin: page 22*) If $z, w, z_i \in \mathbb{C}$ for $i = 1, 2, \ldots, n$ then show that
$$|z_1 + z_2 + \ldots + z_n| \leq |z_1| + |z_2| + \ldots + |z_n|$$
and
$$||z| - |w|| \leq |z - y|.$$
8. (*From Rudin: page 22*) Decide when does the equality hold in Cauchy-Schwarz-Bunyakovski inequality ?

Errata:

1. The set $B = \{p \in \mathbb{Q} : p^2 > 2\}$ was defined. It was supposed to be $B = \{p \in \mathbb{Q} : p > 0, p^2 > 2\}$. The proof indicated in class for showing that there is no smallest element in B will work. Note that in the first definition, the fact that there is no smallest element is trivial.
2. In Rudin Third edition: Theorem 1.21: should read as follows:
"*For every real $x > 0$ and every integer $n > 0$ there is one and only one positive real y such that $y^n = x$*"
In Rudin the word positive is missing. Note that there are readily available counter examples for this.