1. A small college has 1095 students. What is the probability that more than five students were born on Christmas day? Assume that birthrates are constant throughout the year and that each year has 365 days.

2. The length of time (in appropriate units) that a certain type of component functions before failing is a random variable with probability density function

$$f(x) = \begin{cases} 2x & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

   Once the component fails it is immediately replaced with another one of the same type.

   (a) If we let $X_i$ denote the lifetime of the $i^{\text{th}}$ component to be put in use, then $S_n = \sum_{i=1}^{n} X_i$ represents the time of the $n^{\text{th}}$ failure. The long–term rate at which failures occur is

$$r = \lim_{n \to \infty} \frac{n}{S_n}$$

   Determine $r$, assuming that the random variables $X_i$ are independent.

   (b) How many components would one need to have on hand to be approximately 90% certain that the stock would last at least 35 units of time?

3. Two types of coin are produced at a factory: a fair coin and a biased one that comes up heads 55% of the time. We have one of these coins but do not know whether it is a fair or biased coin. In order to ascertain which type of coin we have, we shall perform the following statistical test. We shall toss the coin 1000 times. If the coin comes up heads 525 or more times we shall conclude that it is a biased coin. Otherwise, we shall conclude that it is fair. If the coin is actually fair, what is the probability that we shall reach a false conclusion? What would it be if the coin were biased?

[1] **Office hours:** Monday:2:00-3:00pm, Tuesday:10:30-11:30, 3:30-4:30 pm, Friday: 12:30-1:30pm

**Practice Problems from chapter 5 and 6**

4. The joint distribution of amount of pollutant emitted from a smokestack without a cleaning device ($X_1$) and with a cleaning device ($X_2$) is given by

$$f(x_1, x_2) = \begin{cases} k & \text{if } 0 \leq x_1 \leq 2,\ 0 \leq x_2 \leq 1 \text{ and } 2x_2 \leq x_1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find $E(X_1 \mid X_2 = 2/3)$

   (b) The reduction in amount of pollutant emitted due to the cleaning device is given by $U = X_1 - X_2$.Find the probability density function for $U$. Find $E(U)$. *Solution on the web*

5. *Use method of transformation* Let $X$ be an exponential random varibale with mean 1. Find the probability denisty function for $U = \sqrt{X}$.

6. *Use the Moment Generating function method* Suppose $X$ is Normal$(a, b^2)$ and $Y$ is an independent Normal$(c, d^2)$.

   (a) Let a=c=0, b=d=1. Find the probability density function of average of $X + Y$.

   (b) Do the above for any $a$ and $b$.

7. Let $R$ be the region between $y = x$ and $y = x^2$. A random point $(X, Y)$ is selected from $R$. Find the joint probability density function of $X$ and $Y$.