1. Two random variables X and Y have the following joint probability mass function:

p(x,y)	$X = -1$	$X = 0$	$X = 2$
$Y = 0$	0.0	0.1	0.1
$Y = 1$	0.1	0.2	0.0
$Y = 2$	0.1	0.1	0.3

Find $E(XY)$.

2. Suppose that X and Y are random variables with joint probability density

$$f(x, y) = \begin{cases} \frac{c}{2} & \text{if } 0 \le x \le y \le 2, \\ 0 & \text{otherwise} \end{cases}$$

   (a) Compute c
   (b) Compute $f_X(x), f_Y(y)$,
   (c) Compute $f_{X|Y}(x \mid y)$,
   (d) Calculate μ_X and μ_Y

Ans:

$$c = 1, f_X(x) = \begin{cases} \frac{x}{2} & \text{if } 0 \le x \le 2, \\ 0 & \text{otherwise} \end{cases}, f_Y(y) = \begin{cases} \frac{(2-y)}{2} & \text{if } 0 \le y \le 2, \\ 0 & \text{otherwise} \end{cases}$$

$$f_{X|Y}(x \mid y) = \begin{cases} \frac{xy+1}{y/2+1} & \text{if } 0 \le x, y \le 1, \\ 0 & \text{otherwise} \end{cases}, \mu_X = \mu_Y = \frac{8}{15}$$

3. Suppose X and Y are two continuous random variables. How would you define independence ? Then decide whether the following are independent random variables or not if their joint probability density function is given by:

$$f(x, y) = \begin{cases} 4xy & \text{if } 0 \le x, y \le 1, \\ 0 & \text{otherwise} \end{cases}$$

4. **Tchebysheff's Theorem** (version of Theorem 3.4, page 81) Suppose X is a continuous random variable with probability density function f , then show that, when $a > 0$

$$P(| X | \ge a) \le \frac{E(X^2)}{a^2}$$

the above result is also true for a discrete random variable

[1] **Office hours:** Monday:2:00-3:00pm, Tuesday:10:30-11:30, 3:30-4:30 pm, Friday 1:30-2:30pm