

Math 221 202

(Take home) Quiz 7 Cover-sheet

Score

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Your name \_\_\_\_\_ Student #

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**Due: Thursday, March 8th, 2001 at 10.00am.**

## Ground Rules:

1. Donot pick up a quiz for any other person.
2. Please turn in the quiz in person as you enter the class (no later than 10.15 am).

3. Open book and notes. You may consult anyone you want, but you must write up your own solutions.

4. Show your work. Explain your solutions clearly.

5. When you submit the quiz back on Tuesday, please use this cover sheet as the first page.

6. The grader will choose two problems randomly (page 3) and grade them.

7. Maximum possible score will be 25. There will be 4 points for completion and 1 point for attaching this cover sheet. No points for turning in this cover sheet without the quiz.

It would be nice if you use the space below and the back of this page to answer the problems (page 3).  
It will minimize my/our contributions to global warming.

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<sup>1</sup> **Office hours:** Monday 1:30pm-3pm, 5pm-6pm, Wednesday 2:30pm-3:30pm, Thursday at 1:30pm-3:00pm, Friday 11:30am-12:30pm or by appointment.

Overview of Sections 4.1-4.6 (those not mentioned in 2.9)

1. **Vocabulary List:**

- (a) Coordinate of  $x$  relative to a basis  $B$ .
- (b) Change of coordinates matrix.
- (c) Coordinate map.
- (d) Row space.
- (e) Kernel.
- (f) Range.

2. **Key Concepts:**

- (a) Theorem 7
- (b) Geometric interpretation of the coordinates of  $x$  relative to another basis.
- (c) Geometric interpretation of  $\text{Col}(A)$ ,  $\text{Null}(A)$ ,  $\text{Row}(A)$  as subspaces.
- (d) Proof of Theorem 14

3. **Skills to Master:**

- (a) To compute coordinates of  $x$  relative to another basis.
- (b) To compute basis for the Row space of  $A$ .
- (c) Using Theorem 14 to understand solutions of linear systems.

Homework Set no.	Date	Section	Problems
Homework 17	March 5th, 2001	5.1	1,5,15,19,25,31
Homework 18	March 7th, 2001	5.2	1,9,17,21,27
Homework 19	March 9th, 2001	5.3	1,3,5,15,21,27
Homework 20	March 12th, 2001	5.4	1,3,13,17,21
Homework 21	March 14th, 2001	6.1	1,3,7,13,19,31

This is the schedule that we will be following the rest of the semester. We may fall behind but never go ahead of this.

Date	Section
Feb. 27-Mar. 1	4.6,5.1,5.2
March 6-8	5.3,5.4,6.1
March 13-15	6.2, Review, Exam
March 20-22	6.3,6.4,6.5,6.6
March 27-29	7.1,7.2,1.9
April 2-4	2.7 Review

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**Please Read :**

The following questions can be done with the knowledge from 4.4, 2.9 and 4.6. It is advisable to read your solutions to Quiz 5 (on 2.9) and the ones that I gave out first before you attempt this quiz. This will serve both as a review and emphasise the necessary concepts.

**Quiz 7 (Questions)**

1. Section 4.4: 1,10.
2. Section 4.6: 12,17,25.
3. Let  $A_{6 \times 7}$  be a matrix and  $b_{6 \times 1}$  vector. You are given the following row reduction:

$$(A|b) \rightarrow \left( \begin{array}{cccccc|c} -3 & 0 & -2 & -8 & 0 & 19 & -5 & -2b_2 + b_1 \\ 0 & -3 & -5 & 1 & -3 & -11 & -11 & b_2 - 2b_1 \\ 0 & 0 & 19 & 19 & 0 & -38 & 19 & -3b_3 + b_1 + 7b_2 \\ 0 & 0 & 0 & 0 & 38 & -38 & 38 & 19b_4 - 19b_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 38b_5 - 4b_1 - 9b_2 - 26b_3 + 19b_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 38b_6 - 19b_2 - 19b_4 \end{array} \right)$$

- (a) Is  $y_{6 \times 1} = [2 \ 2 \ -1 \ 0 \ 0 \ 1]^T$  in the  $\text{Col}(A)$  ? Justify your answer.
  - (b) What is the  $\text{rank}(A)$  and  $\text{nullity}(A)$  ?
  - (c) Can you find a matrix  $B$  such that  $\text{Col}(A) = \text{Null}(B)$  ?
4. Let the  $W = \{x_{3 \times 1} \mid 3x_1 + 2x_2 - 4x_3 = 0\}$ .
    - (a) Verify that  $W$  is a subspace of  $\mathbb{R}^3$ .
    - (b) What is  $W$  geometrically ? If  $W = \text{Nul}(A_{m \times n})$  then find  $A_{m \times n}$ .
    - (c) Find a basis  $\mathcal{B}$  for  $W$ .
    - (d) Verify that  $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$  is in  $W$  and find its coordinates relative to  $\mathcal{B}$  that you found in the previous part.
  5. State whether the following are true or false. Please explain your answer.
    - (a) If  $S$  is a subspace of  $\mathbb{R}^3$  then  $\dim(S)$  can be 4.
    - (b) The set  $\{(m, n) \text{ such that } m \text{ and } n \text{ are integers}\}$  is a subspace of  $\mathbb{R}^2$ .
    - (c) If a subspace  $V$  of  $\mathbb{R}^n$  has dimension  $n$ , then  $V = \mathbb{R}^n$ .
    - (d) Let  $T$  be a linear Transformation from  $\mathbb{R}^5$  to  $\mathbb{R}^3$ . Then  $\dim(\text{Kernel}(T))$  has to be 4.
  6. Give an example of a linear transformation  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  such that  $\text{Kernel}(T) = \text{Range}(T)$  (i.e. if  $T(x) = Ax$ , an example of a matrix  $A$  such that  $\text{Null}(A) = \text{Col}(A)$ ). Show that it is not possible to construct a matrix  $A_{3 \times 3}$  such that  $\text{Null}(A) = \text{Col}(A)$ .