Math 221 202 **(Take home) Quiz 5 Cover-sheet** **Score**

Your name ______________________ **Student #** | | | | | | | | |

Due: Tuesday, February 13th, 2001 at 10.00am.

(***Please read carefully***)**Ground Rules:**
0. Donot pick up a quiz for someone else unless they plan to be in class later today.
1. Open book and notes. You may consult anyone you want, but you must write up your own solutions.
2. Show your work. Explain your solutions clearly.
3. When you submit the quiz back on Tuesday, please use this cover sheet as the first page.
4. The grader will choose two problems randomly (page 3) and grade them.
5. Maximum possible score will be 25. There will be 4 points for completion and 1 point for attaching this cover sheet. No points for turning in this cover sheet without the quiz.
6. Please turn in the quiz in person as you enter the class (no later than 10.15 am)

It would be nice if you use the space below and the back of this page to answer the problems (page 3). It will minmize my/our contributions to global warming.

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[1] **Office hours:** Monday 1:30pm-3pm, 5pm-6pm, Wednesday 2:30pm-3:30pm, Thursday at 1:30pm-3:00pm, Friday 11:30am-12:30pm or by appointment.

Overview of Sections 2.3,2.9

1. **Vocabulary List:**
   (a) Section 2.3:
      i. Singular and Non-singular.
      ii. Invertible transformation.
   (b) Section 2.9:
      i. Subspace.
      ii. Column space of A.
      iii. Null space of A.
      iv. Row space of a matrix A. (*Not in the book*)
      v. Range space of A.
      vi. Basis.
      vii. Dimension.
      viii. Nullity, Rank.
2. **Key Concepts:**
   (a) Section 2.3:
      i. Categorizing key equivalent properties of A.
      ii. Theorem 8, 9.
   (b) Section 2.9:
      i. Subspace
      ii. Null and Column spave of a matrix
      iii. Basis, minimal spanning set.
      iv. dimension.
      v. coordinates relative to the basis.
      vi. Theorem 14,15
3. **Skills to Master:**
   (a) Verifying that subsets are subspaces.
   (b) Computing a basis for a subspace. In particular for Null(A) and Col(A).
   (c) Computing rank, nullity.
   (d) identifying coordinates relative to the basis.

Homework Set no.	Date	Section	Problems
Homework Set 12	February 8th	2.9	3,7,17,21,23,25,29,31,35,37,38
Homework Set 13	February 12th	3.1	1,9,15,17,21,25,37,40(b),41,
Homework Set 14	February 15th	3.2	1,3,7,21,23,27,28,29,33
		3.3	7,23,25,27,31,32

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Quiz 5 Questions : Corrected version:

Syllabus: 2.3, 2.9

Solve the following questions:

1. First look at the solution to Q-3(d) in the Triterm and understand it.
   (a) Produce an example of matrices $A_{2\times 3}$ and $B_{3\times 2}$ **not** invertible but AB is invertible.
   (b) Suppose A and B are both invertible then show that AB is invertible.

2. Let v_1, v_2, v_3 be three distinct vectors in $\mathbb{R}^3$. Define the following terms: a vector, a line and a plane in $\mathbb{R}^3$. Keeping these definitions in mind, please answer the following giving reasons:
   (a) Produce an example of v_1 such that span$\{v_1\}$ is a vector.
   (b) Produce an example of v_1 such that span$\{v_1\}$ is a line.
   (c) Produce an example of v_1, v_2 such that span$\{v_1, v_2\}$ is a line.
   (d) Produce an example of v_1, v_2 such that span$\{v_1, v_2\}$ is a plane.
   (e) Produce an example of v_1, v_2, v_3 such that span$\{v_1, v_2, v_3\}$ is a line.
   (f) Produce an example of v_1, v_2, v_3 such that span$\{v_1, v_2, v_3\}$ is a plane.
   (g) Produce an example of v_1, v_2, v_3 such that span$\{v_1, v_2, v_3\} = \mathbb{R}^3$. In this case is it possible to produce a vector v_4 such that $\{v_1, v_2, v_3, v_4\}$ are linearly independent ?

3. Section 2.9: 27,29,38.

4. Look at $A = \begin{pmatrix} 1 & 2 & 0 \\ -1 & 1 & 1 \\ 1 & 0 & 3 \\ 2 & 3 & -2 \end{pmatrix}$, and say $b = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$.
   (a) Are the columns of A linearly independent?
   (b) Is $Ax = b$ consistent?
   (c) " The matrix A is non-singular, because the columns of A are linearly independent. But the system $Ax = b$ is inconsistent, so theorem 8 is false." Is there a problem with this argument ? (explain)

5. Let $M = \begin{pmatrix} 2 & 3 \\ 6 & 9 \end{pmatrix}$
   (a) Find a basis for the Null-space of M and a basis for the Range-space of M.
   (b) Find a 2x2 matrix that has same null space as M but a different range.
   (c) Find a 2x2 matrix that has the same range as M but a different null space.

6. Let $A = \begin{pmatrix} 1 & 1 & -1 & 3 \\ 2 & 0 & 4 & 2 \\ -1 & 1 & -5 & 1 \\ 1 & -1 & 5 & -1 \\ 1 & 2 & -4 & 0 \end{pmatrix}$
   (a) Find a subset of the columns of A that form a basis for Col(A). Find the dimension of Col(A).
   (b) Construct a basis for Col(A) which does not consists of the columns of A.
   (c) Find a basis for and the dimension of $Null(A)$. Confirm that $Rank(A) + Nullity(A)$ is what it is supposed to be.

7. Let W be a subspace of R^3, and let $B = \{e_1, e_2, e_3\}$ be the standard basis of R^3. Since B is linearly independent and since every vector w in W can be written as a linear combination of the vectors in B, it follows that B is a basis for W. Is the above argument correct ? If yes give reasons. else show by example that the argument is not correct.

8. Let $A = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$. Let $b = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Verify Ax=b is INconsistent. Please solve $A^T Ax = A^T b$. (You should get infintely many solutions). Pick any solution and call it $x*$, and let $v = Ax*$. systems.

   Prize Question: (*Need not turn in with the quiz*) What is the geometric significance of the vector v ? If anyone of you solves it, the whole class gets free candy.
   Prize offer expires February 20.