Math 221 **202** **(Take home) Quiz 4 Cover-sheet** **Score**

Your name ______________________ **Student #** | | | | | | | | |

Due: Tuesday, January 30th, 2001 at 10.00am.

(*Please read carefully*)Ground Rules:

0. Donot pick up a quiz for someone else unless they plan to be in class later today.
1. Open book and notes. You may consult anyone you want, but you must write up your own solutions.
2. Show your work. Explain your solutions clearly.
3. When you submit the quiz back on Tuesday, please use this cover sheet as the first page.
4. The grader will choose two out of the 10 problems (page 3) and grade them.
5. Maximum possible score will be 25. There will be 4 points for completion and 1 point for attaching this cover sheet. No points for turning in this cover sheet without the quiz.
6. Please turn in the quiz in person as you enter the class (no later than 10.15 am)

It would be nice if you use the space below and the back of this page to answer the problems (page 3). It will minmize my/our contributions to global warming.

Overview

1. **Vocabulary List:**
   (a) Section 1.7,1.8
      i. Linear transformation
      ii. One to one and onto transformation
      iii. Dialation, contraction, and identity transformation
      iv. Identity Matrix
   (b) Section 2.1,2.2
      i. Transpose.
      ii. Scalar or dot product.
      iii. Matrix addition, multiplication.
      iv. Inverse matrix.
      v. Singular.
2. **Key Concepts:**
   (a) Section 1.7,1.8
      i. Geometric description of the transformation (discussion on page 78/79)
      ii. Standard matrix of transformation
   (b) Section 2.1,2.2
      i. Algebra/Properties of matrices: Addition, multiplication,transpose, and inverse.
      ii. Matrix multiplication and its various interpretations.
      iii. Theorem 7 on page 115
3. **Skills to Master:**
   (a) Section 1.7,1.8
      i. Finding the standard matrix of transformation from the information given.
      ii. Deciding whether a map is a Linear transformation or not.
   (b) Section 2.1,2.2
      i. Add and multiply matrices.
      ii. Algebraic manipulations with matrices.
      iii. Finding the inverse of a matrix.
      iv. Using Theorem 5.
4. **other**
   (a) WWW: Do check out the projects page. There is a link to Section 203's web-page, if you so desire there are more problems for practice. Also a link to Math 152 (an engineering equivalent of 221, which has some neat notes and worked out problems) is worthy of a mouse click.
   (b) It so happens that I designed the syllabus so that we do all the applications at the end of the semester. Sections 1.9,2.7 and 2.8 are easy to read material and cool applications are presented. Do read them at leisure to get a glimpse of how useful the techniques (that we have learnt so far) are.

Homework Set no.	**Date**	**Section**	**Problems**
Homework Set 10	Jan. 27th, 2001	2.2	1,4,5,7,9,10,14,23,32
Homework Set 11	Jan. 30th, 2001	2.3	1,5,9,13,14,15,16,17,21,32

[1] **Office hours:** Monday 1:30pm-3pm, 5pm-6pm, Wednesday 2:30pm-3:30pm, Thursday at 1:30pm-3:00pm, Friday 11:30am-12:30pm or by appointment.

Quiz 4

Syllabus:1.7,1.8,2.1,2.2
Solve the following questions:
1. Section 1.7: 13,
2. Section 1.8: 13,24,
3. Section 2.1: 5,23,
4. Section 2.2: 7
5. Suppose T is a linear transformation and

$$T\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \quad T\left(\begin{bmatrix} 3 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 2 \\ 2 \\ 1 \end{bmatrix},$$

Find a matrix A such that $T(x) = Ax$ for all $x \in \mathbb{R}^2$.

6.For parts (a) and (b), please justify your answer by giving reasons.
a) Suppose A is a 2×3 matrix such that each entry has absolute value ≤ 5. That is, $-5 \leq a_{ij} \leq 5$ where a_{ij} refers to the scalar in row i and column j of the matrix A.

i) Is it possible that $Ae_2 = \begin{bmatrix} 10 \\ 1 \end{bmatrix}$ where $e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$?

ii)Is it possible that $Av = \begin{bmatrix} 10 \\ 1 \end{bmatrix}$ for some $v \in \mathbb{R}^3$?

b)Does there exist a Linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^1$ such that

$$T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = [\ 1\ ], \quad T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = [\ 2\ ], \quad T\left(\begin{bmatrix} 2 \\ 0 \end{bmatrix}\right) = [\ 4\ ]$$

7. Let $Q^{-1} = \begin{pmatrix} 1 & 1 & -1 \\ -1 & -1 & 2 \\ 1 & 0 & -1 \end{pmatrix}$ and $G^{-1} = \begin{pmatrix} -1 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. Find a vector x such that $GQx = \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix}$, and check your answer.

8. Let $I_{2\times 2}$ be the identity matrix, $A^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} -2 & 3 \\ 1 & 5 \end{bmatrix}$.
Using matrix operations compute the matrix Q when

$$Q = ((3AB)^T)^{-1} + A^{-1}(2A + I) + B^{-1}.$$

9. (No need to turn in) Please read the discussion on page 78,79,80. I did not draw too many pictures for Linear transformations. A big part about them is how to picture them.

Quote of the week: *"You better cut the pizza in four pieces because I'm not hungry enough to eat six."* -*Yogi Berra*

Triterm I: February 1, 2001.
Syllabus: 1.1-1.8, 2.1-2.3.

1. **Exam Instructions:**
   (a) Be prepared to show ID during the test.
   (b) The Exam will be closed book, no notes.
   (c) A simple calculator will be allowed...(i.e. one without programmable function of any kind and any other fancy stuff). However the exam will be so designed that you will not need a calculator.
   (d) Please show your work in the exam, write in complete sentences, and Indicate your answers clearly.
   (e) The exam will have 5 or 6 questions.
2. In the exam you may be asked to state the definitions of any of the following terms:
   (a) Linear system of Equations.
   (b) Three elementary operations.
   (c) Row equivalence of matrices.
   (d) Echelon form and Reduced echelon form.
   (e) Linear combinations of vectors.
   (f) Span.
   (g) Linear dependence and Independence.
   (h) Linear Transformation.
   (i) Singular and Non-Singular matrices.
   (j) Identity matrix and the Inverse of a matrix.
3. **Studying:** Begin by reviewing the overview sheets, the solved examples in the text, the homework problems, the quiz questions and the practice problems given below. Remember while doing the problems make sure you to understand how each of them fits into the section/chapter. Finally try as many problems from the text as you can.

Some practice problems: *Questions* $1-5$, *along with Q5 in quiz 4 could be treated like a sample exam. One way to test yourself is to study and see how many questions you can answer, than repeat the process till you finally solve all the questions.*

1. Let

$$A = \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B^T = \begin{bmatrix} -1 & 3 \\ 1 & 4 \end{bmatrix}, \quad \text{and} \quad v = \begin{bmatrix} 9 \\ -7 \end{bmatrix}.$$

(a) Calculate BA^T

(b) Find $(-A + (A^T(B+I))^T - B^T A)v$
(Hint: Simplify using matrix operations)

2. For the following, find the correct response. Give **reasons** for your answer

(a) One can obtain $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ from $\begin{bmatrix} 3 & 5 \\ 4 & 1 \end{bmatrix}$
by elementary row operations. True False

b) Assume A is a singular $n \times n$ matrix. $b \in R^n \neq \theta$
(i) Then $Ax = b$ is consistent. Possible Impossible
(ii) Then $Ax = b$ has a unique solution. Possible Impossible

(c) $A^T A$ is not symmetric for some $n \times n$ matrix A Possible Impossible

[2] **Office hours:** Monday 1:30-3pm, 5-6pm, Wednesday 2:30-3:30pm, Thursday 1:30-3:00pm, Friday: 11:30-12:30 or by appointment.

d) $A_1, A_2, \ldots, A_n$ are n column vectors $\in R^n$. $v \in R^n$ and v is a linear combination of $A_1, \ldots, A_n$. A is the $n \times n$ matrix with columns $A_1, \ldots, A_n$

consistent (i) Then $Ax = v$ is consistent. True False

(ii) Then $A_1, A_2, \ldots, A_n, v$ are linearly independent. Possible Impossible

(e) A is a 3×3 matrix. B is a 3×4 matrix. Then the columns of AB are linearly independent. Possible Impossible

If the following are Augmented matrices for systems of equations, state whether or not the systems are consistent or inconsistent. If , state the number of unconstrained variables.

(f)

$$\begin{bmatrix} 0 & 0 & 2 & 2b \\ 0 & 0 & 1 & b \\ 0 & 1 & 0 & a \end{bmatrix}$$

a, b some scalars.

(g)

$$\begin{bmatrix} 1 & 2 & 1 & 3 & 0 \\ 0 & 0 & -3 & -4 & 1 \\ 0 & 0 & 3 & 4 & -1 \end{bmatrix}$$

3. Consider The linear system:

$$\begin{array}{rcrcrcr} 3x_1 & + & 2x_2 & + & x_3 & = & 10 \\ x_1 & + & x_2 & + & x_3 & = & 6 \\ x_1 & - & 2x_2 & + & x_3 & = & 0\,. \end{array}$$

(a) Write the augmented matrix corresponding to this system.

(b) Reduce the augmented system in part (a) to echelon form.

(c) Describe the set of solutions to the given system.

4.

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 0 \\ -3 \\ -2 \end{bmatrix}, \quad \text{and} \quad b = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix},$$

If possible, write b as a linear combination of v_1, v_2, v_3.

5.(a)

$$A = \begin{bmatrix} 1 & 4 & 2 \\ 0 & 2 & 1 \\ 3 & 12 & 7 \end{bmatrix}$$

Compute A^{-1}

(b) Let B be a 40×40 matrix such that $B^2 = 0$. Use the definition of the Inverse and properties of matrix Arithmetic to show that $I - B$ is the inverse of $I + B$.