Math 105 91A[1] January 31st, 19100 Worksheet 5

http://www.math.ubc.ca/~ athreya/105

1. Using the method of substitution find the following integrals.
   (a) $\int \frac{3}{x-1} + \frac{6}{\sqrt{x}(\sqrt{x}-1)^3} dx$
   (b) $\int a^t dt$ where a is some constant.
   (c) Do 51 and 71 from Exercise 16.3.

2. Find the total revenue r from selling q units if the marginal revenue in dollars per unit for $0 < q \leq 400$ is given by $\frac{dr}{dq} = \frac{125}{\sqrt{x}} - \frac{100}{x^2}$ and the revenue from selling the first 100 units is \$2400. What is the additional income that comes from selling the next 100 units ?

3. If the rate of change of the value V of a house that cost $\$350,000$ to build is given by $8e^{0.05t}$, where t is the time in years since the house was built. Find out how long it will take for the house to double.

[1] **Office hours:** Monday:2:30-4:00pm, Tuesday:11:30-12:30 am, 3:30-4:30 pm, Wednesday: 10:30-11:30 am

Problems from chapter 19.

1. Let $f(x, y) = 2x^2 - 3xy + 4y^2 + ax + by$, where a and b are constants. It is known that $f(x, y)$ attains a local maximum at $x = 2, y = 3$. Find a and b.

2. Let $f(x, y) = \ln(x^2 + y^2)$. Compute $f_{xx} + f_{yy}$. Please show your work and simplify. sweat-shop

3. Let $f(x, y) = xy - \frac{x^4}{2} - y^2$. Where does $f(x, y)$ attain a maximum ? (Assume that $f(x, y)$ does attain a maximum. There is no need to use the second derivative test)

4. In a country with business-friendly labour laws, a sweat-shop is billing an order for 120,000 Maple Leaf sweat shirts. By using x units of labour and y units of capital, the sweat-shop can produce $x^{\frac{2}{3}}y^{\frac{1}{3}}$ shirts. The cost per unit of labour is \$0.25. Use the method of Lagrange multipliers to find how much it costs to produce the sweat-shirts if the minimizes the cost of production.

5. In economics, a Cobb-Douglas production function is a production function of the form $P = Al^{\alpha}k^{\beta}$, where l and k denote the number of units of labour and capital respectively and A, α and β are constants and $\alpha + \beta = 1$. For such a function, (a) Find the marginal productivity with respect to k. (b) Show that $l\frac{\partial P}{\partial k} + k\frac{\partial P}{\partial l} = P$. (c) If the labour and the capital are each increased by 50%, by how much does the production increase ?

6. Find the relative extrema and saddle points of f defined by $f(x, y) = x^2 - 4xy + y^3 + 4y$.

7. Suppose a firm owns two production plants X and Y. The cost of producing x items from plant X is $x(3x + 4)$ hundreds of dollars, while the cost of producing y items from plant Y is $y(2y + 8)$. Determine how many items should be produced at each plant to fill an order of 149 items and minimize the cost. (Use the method of Lagrange multipliers)

8. A manufacturer produces two products, X and Y. Let $C = f(x, y)$ be this manufacturer total cost function, where x any y denote the numbers of items of X and Y produced. Assume that $f(30, 25) = 20000$, $\frac{\partial f}{\partial x}(30, 25) = 10$ and that $\frac{\partial f}{\partial y}(30, 25) = 15$. (a) On the basis of this information, approximate $f(32, 25)$. (b) Approximate $f(32, 22)$.

9. Let $P(l, k)$ be a production function, where l and k denote the number of units of labour and capital respectively. Assume that we need to respect the budget constraint $C = al + bk$, where C is our total budget, and a and b denote the cost of one unit of labour and capital respectively. Show that when the production is maximized, $\dfrac{\frac{\partial P}{\partial l}}{a} = \dfrac{\frac{\partial P}{\partial k}}{b}$

10. Let $p = f(l, k, d)$ be a production function where l, k and d denote the number of units of labour, capital and land respectively. Assume $p(50, 60, 40) = 200,000$, $\frac{\partial p}{\partial l}(50, 60, 40) = 100$, $\frac{\partial p}{\partial k}(50, 60, 40) = 200$, and $\frac{\partial p}{\partial d}(50, 60, 40) = 50$. On the basis of this information approximate $p(49, 62, 38)$.

11. The total cost of making x items at factory X and y items at factory Y is given by $C(x, y) = 2x^2 + xy + y^2 + 500$. Assume that is desired to make a total of 200 items. (a) Use Lagrange multipliers to find the values of x and y which minimize the total cost. (b) Use any other calculus based method to find the values of x and y which minimizes the total cost and check that your answers agree with the answers to part (a).