Mathematics 105 Final Exam Guide

On the April final examination, you may bring in an 8.5 inch by 11 inch information sheet, and use any calculator(s) that you wish to. Please make sure that you can answer questions that deal with the following items.

1. Use the basic rules of differentiation to find partial derivatives and recognize that such derivatives are instantaneous rates of change when all but one of the independent variables are held fixed. Be familiar with Cobb-Douglas production functions and their associated marginal productivities of labour and capital as important special cases.

2. Solve max/min problems without constraints by setting the partial derivatives equal to 0 and then solving the resulting equations for the critical points. You will *not* be required to use the second derivative test to justify that your answer is a max/min.

3. Use the method of Lagrange Multipliers to solve max/min word problems with constraints. A problem on the final exam may require you to use Lagrange Multipliers even though the problem can be transformed so that it can be solved without Lagrange Multipliers.

4. Find an antiderivative of a function $f(x)$ by observation, substitution, integration by parts, simple partial fractions (involving two linear factors in the denominator of a rational function) or a combination of these methods. Remember that your final answer can be checked by taking the derivative. Understand that an indefinite integral denotes all possible antiderivatives of a function and involves an arbitrary constant.

   In your review of antiderivatives remember the basic ones that are missing from the textbook, namely

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C,$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C \qquad \int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C.$$

5. Solve word problems where you are given the instantaneous rate of change of a function, and are asked to find the change of the function over a particular time interval (possibly infinite). That is, use the definite integral to find the change of a function.

6. Solve word problems where you are given the instantaneous rate of change of a function and the value of the function at a particular value of the independent variable, and are asked to find the function, or possibly the value of the function at some other value of the independent variable.

7. Find the area between two curves by using definite integrals. One of the curves could be the x-axis, and the curves could intersect at more than two points.

8. Use integration (proper and improper integrals) in problems involving the present or future value of a continuous income stream.

9. Use integration to find consumers' or producers' surplus. Be able to make these calculations with either p or q as the independent variable.

10. Use the Trapezoidal Rule to approximate the value of a definite integral in case the antiderivative is impossible to find, or when the function is only given at a finite number of values of the independent variable. You will not be asked to use Simpson's Rule. You may be asked to approximate a definite integral by rectangles to test whether you understand the definition of the definite integral as a limit of such rectangular approximations (Riemann Sums).

11. Solve simple first-order differential equations with initial conditions by separation of variables.

12. Use tree diagrams to solve word problems in probability.

13. Use data in tables to find probabilities.

14. Solve word problems involving binomial and hypergeometric distributions. That is, solve word problems involving sampling with and without replacement.

15. Solve word problems leading to discrete and continuous random variables with a view to finding the associated probability distribution functions or density functions. Once the probability distribution function or density function is known, probabilities can be calculated by summation or integration.

16. Find the expected value (mean, expectation), variance, and standard deviation of a discrete or continuous random variable. These calculations could arise in word problems or you could simply be given the probability distribution or density function. In the density case, this type of problem reduces to an exercise in integration.