Math 104 90A[1] October, 13th 1999. Extra Credit Worksheet

http://www.math.ubc.ca/~ athreya/104 **Due: November, 24th 1999**

Game rules: You may consult anyone/any textbook that you want to. However, you must write up your own solutions. The maximum score on this worksheet is 25 points. There will 3 points for clarity 3 points for effort and 1 point for neatness.

Extra credit: Let $x \leq y \leq z$ be your three scores from the Triterms. Let a be your score from this worksheet. The lowest Triterm score x will be replaced with $x_1 =$ minimum of $x + a$ and $(y + z)/2$. Your term grade will be calculated with the Triterm scores x_1, y and z.

Mean Value Theorem and Intermediate Value Theorem

There are two Theorems which are fundamental in Calculus and are not covered in this course. In this worksheet we shall get to know them and their applications.

Mean Value Theorem: Let $a < b$ be two real numbers. Suppose $f : \mathbb{R} \to \mathbb{R}$ satisfies the following conditions:

(C) f is a continuous function in the closed interval $[a, b]$.

(D) f is a differentiable function in the open interval (a, b).

then there exists a point $c \in (a, b)$ such that

$$f(b) - f(a) = f^{'}(c)(b - a). \tag{1}$$

Solve the following questions:

1. (2 points) Describe the Mean value Theorem geometrically.

2. (4 points) The Mean Value Theorem says that the conditions (C) and (D) are sufficient for (1) to hold. Are conditions (C) and (D) necessary for (1) to hold ? Justify your answer.

3. (4 points) Suppose you do a road trip to Seattle from Vancouver. It takes you 3 hours and 30 minutes to cover the distance of 240 km.

[1] **Office hours:** Monday: 1:45-3:15pm, Tuesday: 3:30-5:30pm, Wednesday 2:15-3:15pm

(a) What was your average velocity ?

(b) Assume there was no sudden stops and jerks on your trip. Is there a time at which your instantaneous velocity was equal to your average velocity ? Justify.

Intermediate Value Theorem Let $a < b$ be two real numbers. Suppose $f : \mathbb{R} \to \mathbb{R}$ is a continuous function and $f(a) < f(b)$. Then for any real number y such that

$$f(a) < y < f(b)$$

there is a real number c such that $a < c < b$ and

$$y = f(c)$$

Solve the following questions:

1. (2 points) Describe the Intermediate Value Theorem geometrically.
2. (2 points) Does the Theorem hold for functions f that are not continuous ? Justify.
3. (4 points) In Vancouver the temperature is 20° Celsius. You draw a circle going north on the globe starting and ending at Vancouver. collect temperature data on a few selected places. You find the temperature at one point on the curve to be -20° Celsius and temperature at another to be 30° Celsius. Is there another point in the world that has the same temperature as Vancouver ? Justify.