1. Find the error[2] in the proofs below if any. Please rewrite the proof precisely.

Result: $\lim_{n\to\infty} \frac{2-n}{1+n^2+n} = 0$

Proof: For all $\epsilon > 0$ by Archimedean Property on $\mathbb{R}$ there is a $N \in \mathbb{N}$ such that $N > 2-\epsilon$. For any $n > N$ we have

$$\begin{aligned} a_n &= \frac{2-n}{1+n^2+n} \\ &< \frac{2-n}{1} \\ &\le 2-N \\ &< \epsilon. \end{aligned}$$

Thus for all $n \ge N$, $\mid a_n - 0 \mid < \epsilon$ for all $\epsilon > 0$. Since $\epsilon > 0$ was arbitrary, we are done. □

Result: $\lim_{n\to\infty} \frac{1+n}{1+n^2+n} = 0$

Proof: Let $\epsilon > 0$ be given. By Archimedean Property on $\mathbb{R}$ there is a $N \in \mathbb{N}$ such that $\frac{1}{N} < 2\epsilon - 1$. For any $n > N$ we have

$$\begin{aligned} \mid a_n \mid &= \frac{1+n}{1+n^2+n} \\ &< \frac{1+n}{2n} \\ &= \frac{1}{2n} + \frac{1}{2} \\ &= \frac{1}{2N} + \frac{1}{2} \\ &< \frac{1}{2}(2\epsilon - 1) + \frac{1}{2} = \epsilon. \end{aligned}$$

Thus for all $n \ge N$, $\mid a_n - 0 \mid < \epsilon$. Since $\epsilon > 0$ was arbitrary, we are done. □

2. Let $\{x_n\}_{n=1}^{\infty}$be a sequence of real numbers. $a \in \mathbb{R}$ is said to be a limit point of $\{x_n\}_{n=1}^{\infty}$if every interval around a contains infinitely many $\{x_n\}_{n=1}^{\infty}$. Write an equivalent statement of in logical notation. Then write a logical statement that is equivalent to the statement "$b \in \mathbb{R}$ is not a limit point of $\{x_n\}_{n=1}^{\infty}$."

3. Find the limit points of $\{x_n\}_{n=1}^{\infty}$when they are given by
   (a) $x_n = \frac{(-1)^n n}{n+1}$
   (b) $x_n = a^{\frac{1}{n}}$ for $0 < a < 1$
   (c) $x_n = \sum_{k=1}^{n} r^k$, $0 < r < 1$.

4. Find the largest and the smallest limit points of $\{x_n\}_{n=1}^{\infty}$when they are given by
   (a) $x_n = (-1)^n + (-1)^{n+2}$
   (b) $x_n = 2(-1)^n + \frac{n}{n+1}$
   (c) $x_0 \in \mathbb{R}$ and for $n \ge 1$,
   $$x_n = \frac{1}{2} x_{n-1}$$
   (d) $0 < x_n < \frac{1}{n}$

[1]**Office hours:** I will be in my office from 9am-10am Monday, 8am-9am Tue and Thu, 10:00-11:00am Tue to answer any questions that you may have. Please feel free to drop by during these times to clarify any doubts that you may have.
[2]Credits: problem provided by Dr. Purvi Gupta