3 (a)

last class

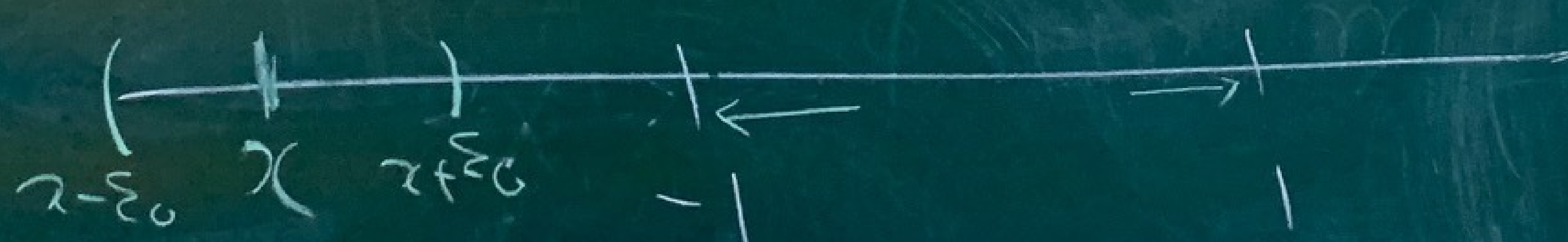
-1 is a limit point

1 is a limit point

$x \notin \{-1, 1\}$ it is not a limit point

$$x_n = \begin{cases} \frac{n}{n+1} & \text{if } n\text{-even} \\ -\frac{n}{n+1} & \text{if } n\text{-odd} \end{cases}$$

$$= \begin{cases} 1 - \frac{1}{n+1} & \text{if } n \text{ is even} \\ -1 + \frac{1}{n+1} & \text{if } n \text{ is odd} \end{cases}$$



- $x < -1$ let $\varepsilon_0 = \frac{|x-(-1)|}{2}$ $(x-\varepsilon_0, x+\varepsilon_0) \not\ni x_n \;\forall n \geq 1$

let

There is no continuous bijection from $(0,1) \to [0,1]$

<u>Sketch</u>: If $\exists\, f: (0,1) \to [0,1]$ continuous bijection

1. Show that f is increasing or decreasing

2. $\exists\, x_0 \in (0,1)$ $f(x_0) = 1$

   $\exists\, y_0 \in (0,1)$ $f(y_0) = 0$

Assume $f : [0,1] \to (0,1)$ is a continuous bijection.

Construct $\{x_n\}_{n \in \mathbb{N}}$ as

$\exists\, y \in [0,1] \ni f(y) = 1/2 \in (0,1)$. Let $x_1 = y$.

Suppose we have assigned a value to $x_n \ \forall\, n \leq k$.

Assign x_{k+1} as

choose $y \in [0,1] \ni f(y) = \frac{1}{k+1} \in (0,1)$

Let $x_{k+1} = y$

By induction, $\{x_n\}_{n \in \mathbb{N}}$ has been constructed

$f(x_n) = \frac{1}{n} \ \forall\, n \geq 2 \Rightarrow f(x_n) \to 0$ as $n \to \infty$.

But $\{x_n\}_{n \geq 1}$ is bounded, so it has a convergent subsequence $\{x_{n_k}\}_{k \in \mathbb{N}}$

$\therefore x_{n_k} \to c \in [0,1]$ as $k \to \infty$

$\therefore f(x_{n_k})$ converges to $f(c)$. $[\because f$ is continuous$]$

$\Rightarrow f(c) = 0 \notin (0,1)$. Contradiction.

Construction of Number Systems — Mohan kumar

We start with Peano's Axium

Peano's Axioms $\mathbb{N}$ is a set

with the following properties

(1) $\mathbb{N}$ has a distinguished element which we call 1

(2) There exists a distinguished set map $\sigma : \mathbb{N} \to \mathbb{N}$

(3) σ is one-to one [injective]

(4) There does not exist an element $n \in \mathbb{N}$ such that $\sigma(n) = 1$. [Not surjective]

(5) let $S \subseteq \mathbb{N}$ such that a) $1 \in S$ and b) if $n \in S$ then $\sigma(n) \in S$. Then $S = \mathbb{N}$ [Principle of Mathematical Induction

We call such a set $\mathbb{N}$ to be set of natural numbers and elements to be natural numbers.

Define $\oplus$

$$n+1 := \sigma(n)$$
$$n+\sigma(m) := \sigma(m+n)$$

$\odot$

$$n \cdot 1 = n$$
$$n \cdot \sigma(m) := n \cdot m + n$$

addition $n+m$ and multiplication $n \cdot m$

Associativity $+,\cdot$ Distributive $\cdot,+$ Commutativity $+,\cdot$	Cancellation $x+z = y+z \Rightarrow x = y$

Order on $\mathbb{N}$:

$n < m$ if $\exists k \in \mathbb{N}$ $m = n+k$.

$n \leq m$ if $n = m$ or $n < m$

Well-ordering

$n, m \in \mathbb{N} \Rightarrow$ $n < m$ or $n = m$ or $n > m$.

$S \subseteq \mathbb{N}$ $\exists$ unique least element of S.