

Name: _____

Score:

1. Let $x_n = \begin{cases} 3 + \frac{1}{n} & \text{if } n \text{ is odd} \\ -6 - \frac{1}{n} & \text{if } n \text{ is even} \end{cases}$

Fill in the boxes:

1. The set of limit points $E = \boxed{}$.

2. $S_k = \sup\{x_n : n \geq k\} = \boxed{}$ and $\lim_{k \rightarrow \infty} S_k = \boxed{} = \boxed{}(E)$.

3. $I_k = \inf\{x_n : n \geq k\} = \boxed{}$ and $\lim_{k \rightarrow \infty} I_k = \boxed{} = \boxed{}(E)$

4. Fix $\epsilon = 0.001$

(a) If we take $N = \boxed{}$ then for $n \geq N$, $x_n < 3 + \epsilon$

(b) Let $M \in \mathbb{N}$. If we take $m = \boxed{}$ then $m \geq M$ and $x_m > 3 - \epsilon$

From (a) and (b) we can conclude that

(i) For all $\epsilon > 0$ many $n \in \mathbb{N}$ we have $x_n \in (0, \epsilon)$.

(ii) For $\boxed{}$ many $n \in \mathbb{N}$ we have $x_n \boxed{} 3 - \epsilon$

5. Fix $\epsilon = 0.005$

(a) If we take $N = \boxed{}$ then for $n \geq N$, $x_n > -6 - \epsilon$

(b) Let $M \in \mathbb{N}$. If we take $m =$ then $m \geq M$ and $x_m < -6 + \epsilon$

From (a) and (b) we can conclude that

(i) For $\boxed{}$ many $n \in \mathbb{N}$ we have $x_n \boxed{} - 6 + \epsilon$

(ii) For all $\boxed{}$ many $n \in \mathbb{N}$ we have $x_n \boxed{} - 6 - \epsilon$

¹No justification is required but please do all rough/fair work on the sheet and the backside