1. Let $x_n = \begin{cases} 3 + \frac{1}{n} & \text{if } n \text{ is odd} \\ -6 - \frac{1}{n} & \text{if } n \text{ is even} \end{cases}$

Fill in the boxes:

1. The set of limit points $E =$ {3,-6}.

2. $S_k = \sup\{x_n : n \geq k\} =$ if $k$ is odd then $3 + \frac{1}{k}$; if $k$ is even then $3 + \frac{1}{k+1}$ and $\lim_{k\to\infty} S_k =$ 3 $=$ sup$(E)$.

3. $I_k = \inf\{x_n : n \geq k\} =$ if $k$ is odd then $-6 + \frac{1}{k+1}$; if $k$ is even then $-6 + \frac{1}{k}$ and $\lim_{k\to\infty} I_k =$ -6 $=$ inf$(E)$

4. Fix $\epsilon = 0.001$
   (a) If we take $N =$ 1000 then for $n \geq N$, $x_n < 3 + \epsilon$
   (b) Let $M \in \mathbb{N}$. If we take $m =$ $2M + 1$ then $m \geq M$ and $x_m > 3 - \epsilon$

   From (a) and (b) we can conclude that

   (i) For all but finitely many many $n \in \mathbb{N}$ we have $x_n$ $<$ $3 + \epsilon$
   (ii) For infinitely many many $n \in \mathbb{N}$ we have $x_n$ $>$ $3 - \epsilon$

5. Fix $\epsilon = 0.005$
   (a) If we take $N =$ 200 then for $n \geq N$, $x_n > -6 - \epsilon$
   (b) Let $M \in \mathbb{N}$. If we take $m =$ 2M then $m \geq M$ and $x_m < -6 + \epsilon$

   From (a) and (b) we can conclude that

   (i) For infinitely many $n \in \mathbb{N}$ we have $x_n$ $<$ $-6 + \epsilon$
   (ii) For all all but finitely many many $n \in \mathbb{N}$ we have $x_n$ $>$ $-6 - \epsilon$

[1] No justification is required but please do all rough/fair work on the sheet and the backside