Due Date: October, 17th 2019

I. Let $\{x_n\}_{n=1}^{\infty}\{y_n\}_{n=1}^{\infty}$and $\{z_n\}_{n=1}^{\infty}$be three sequences. Let $z_n = x_n + y_n$. Let E, F, G be the limit points of $\{x_n\}_{n=1}^{\infty}$, $\{y_n\}_{n=1}^{\infty}$, and$\{z_n\}_{n=1}^{\infty}$respectively. Find the sets in each of the following cases:

Then please fill the following tables:

(i) when $x_n = \frac{1}{n}$, $y_n = (-1)^n$ for all $n \geq 1$.

$E =$	$F =$	$G =$
$\sup(E) =$	$\sup(F) =$	$\sup(G) =$
$M_k = \sup\{x_n : n \geq k\} =$	$N_k = \sup\{y_n : n \geq k\} =$	$O_k = \sup\{z_n : n \geq k\} =$
$\lim_{k\to\infty} M_k =$	$\lim_{k\to\infty} N_k =$	$\lim_{k\to\infty} O_k =$

(ii) when $x_n = 3$, $y_n = -3$ for all $n \geq 1$.

$E =$	$F =$	$G =$
$\sup(E) =$	$\sup(F) =$	$\sup(G) =$
$M_k = \sup\{x_n : n \geq k\} =$	$N_k = \sup\{y_n : n \geq k\} =$	$O_k = \sup\{z_n : n \geq k\} =$
$\lim_{k\to\infty} M_k =$	$\lim_{k\to\infty} N_k =$	$\lim_{k\to\infty} O_k =$

(iii) when $x_n = \begin{cases} 3 + \frac{1}{n} & \text{if } n \text{ is odd} \\ -6 - \frac{1}{n} & \text{if } n \text{ is even} \end{cases}$ and $y_n = \begin{cases} -10 + \frac{1}{n} & \text{if } n \text{ is odd} \\ 3 - \frac{1}{n} & \text{if } n \text{ is even} \end{cases}$

$E =$	$F =$	$G =$
$\sup(E) =$	$\sup(F) =$	$\sup(G) =$
$M_k = \sup\{x_n : n \geq k\} =$	$N_k = \sup\{y_n : n \geq k\} =$	$O_k = \sup\{z_n : n \geq k\} =$
$\lim_{k\to\infty} M_k =$	$\lim_{k\to\infty} N_k =$	$\lim_{k\to\infty} O_k =$

1. Can you conclude any relationship between $\sup(E), \sup(F)$ and $\sup(G)$?
2. Can you provide a proof of the relationship in general ?

[5] **Office hours:** I will be in my office from 9am-10am Monday, 8am-9am Tue and Thu, 10:00-11:00am Tue to answer any questions that you may have. Please feel free to drop by during these times to clarify any doubts that you may have.

II. For any $\{a_n\}_{n=1}^{\infty}$, we wish to understand the behaviour of $\frac{a_{n+1}}{a_n}$ and $a_n^{\frac{1}{n}}$.

Fill the following table when:

(a) $a_n = \frac{1}{2^{2k-1}}$ if $n = 2k-1$, for $k \geq 1$ and $a_n = \frac{1}{2^{2k-2}}$ if $n = 2k$, for $k \geq 1$.

$\frac{a_{n+1}}{a_n}$ when n is odd =	$a_n^{\frac{1}{n}}$ when n is odd =
$\frac{a_{n+1}}{a_n}$ when n is even =	$a_n^{\frac{1}{n}}$ when n is even =
E= The set of limit points of $\frac{a_{n+1}}{a_n}$ =	F= The set of limit points of $a_n^{\frac{1}{n}}$ =
$\sup(E)$ = $\inf(E)$ =	$\sup(F)$ = $\inf(F)$ =

(b) $a_n = \frac{1}{2^k}$ if $n = 2k-1$, for $k \geq 1$ and $a_n = \frac{1}{3^k}$ if $n = 2k$, for $k \geq 1$.

$\frac{a_{n+1}}{a_n}$ when n is odd =	$a_n^{\frac{1}{n}}$ when n is odd =
$\frac{a_{n+1}}{a_n}$ when n is even =	$a_n^{\frac{1}{n}}$ when n is even =
E= The set of limit points of $\frac{a_{n+1}}{a_n}$ =	F= The set of limit points of $a_n^{\frac{1}{n}}$ =
$\sup(E)$ = $\inf(E)$ =	$\sup(F)$ = $\inf(F)$ =

(c) $a_n = \begin{cases} n & \text{if } n = 2k-1, k \geq 1 \\ \frac{1}{3^n} & \text{if } n = 2k, k \geq 1 \end{cases}$.

$\frac{a_{n+1}}{a_n}$ when n is odd =	$a_n^{\frac{1}{n}}$ when n is odd =
$\frac{a_{n+1}}{a_n}$ when n is even =	$a_n^{\frac{1}{n}}$ when n is even =
E= The set of limit points of $\frac{a_{n+1}}{a_n}$ =	F= The set of limit points of $a_n^{\frac{1}{n}}$ =
$\sup(E)$ = $\inf(E)$ =	$\sup(F)$ = $\inf(F)$ =

1. Can you conclude any relationship between $\limsup_{n\to\infty} a_n^{\frac{1}{n}}$ and $\limsup_{n\to\infty} \frac{a_{n+1}}{a_n}$?
2. Can you provide a proof of the relationship in general ?

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