Due Date: October, 10th 2019
Problems Due: 2, 3

1. Find the limit points of $\{x_n\}_{n=1}^{\infty}$with $x_n = \{\alpha n\}$ where $\alpha \in \mathbb{R}$ and $\{x\} :=$ is the fractional part of x.
2. For $\{x_n\}_{n=1}^{\infty}$given in each of the following, please compute $\frac{x_{n+1}}{x_n}$ and determine if the sequence $\frac{x_{n+1}}{x_n}$ has a limit point $a \in \mathbb{R}$ which is greater than or equal to 1.
   (a) $x_n = \frac{1}{n}$
   (b) $x_n = \frac{2^n}{n!}$,
   (c) $x_n = nb^n$, for $b \in (0,1)$
   (d) $x_n = \frac{n!}{n^n}$
   (e) $x_n = \frac{n!}{n^n}$
   (f) $x_n = \frac{(n!)^2}{n^n}$
   (g) $x_n = \frac{2n!}{n^{2n}}$
   (h) $x \in \mathbb{R}$ and for $n \geq 0$, $x_n = \frac{x^n}{n!}$
   (i) $x_n = n^{\frac{1}{n^2}}$
3. For $\{y_n\}_{n=1}^{\infty}$given in each of the following, please compute $y_n^{\frac{1}{n}}$ and determine if $y_n^{\frac{1}{n}}$ has a limit point $a \in \mathbb{R}$ which is greater than or equal to 1.
   (a) $y_n = \left(\frac{n}{2n+1}\right)^n$
   (b) $y_n = \frac{2^n}{n!}$
   (c) $x \in \mathbb{R}$ and for $n \geq 0$, $y_n = n^n x^n$
   (d) $y_n = \begin{cases} \frac{1}{2^k} & \text{if } n = 2k-1, k \geq 1 \\ \frac{1}{3^k} & \text{if } n = 2k, k \geq 1 \end{cases}$
   (e) $y_n = \begin{cases} \frac{1}{2^{2k-1}} & \text{if } n = 2k-1, k \geq 1 \\ \frac{4}{2^{2k}} & \text{if } n = 2k, k \geq 1 \end{cases}$

Puzzle : In G-25, Siva has two bags, 50 red balls and 50 black balls. He distributes the balls between the two bags in an arbitrary manner. Yogesh walks into the room : chooses a bag at random and then chooses a ball at random from the bag. He draws a red ball. Yogesh then asks Siva the following question: *Can you find the arrangement of the balls in the two bags so that when Anita chooses a bag at random and then a ball at random from the balls in the chosen bag, the probability that she chooses a black ball is maximized ?*

[3]**Office hours:** I will be in my office from 9am-10am Monday, 8am-9am Tue and Thu, 10:00-11:00am Tue to answer any questions that you may have. Please feel free to drop by during these times to clarify any doubts that you may have.