Due Date: October, 3rd 2019
Problems Due: 1,3

1. Using the method of induction prove that

   (a) $\sum_{i=0}^{n} i!i = (n+1)! - 1$

   (b) For all $x > 0$, $(1+x)^n \geq 1 + nx$

   (c) Suppose for $1 \leq i \leq n$, a_i are real numbers. Show that

   $$|\sum_{i=1}^{n} a_i| \leq \sum_{i=1}^{n} |a_i|$$

2. Using induction, prove that an n-element set has 2^{n1} subsets with an even number of elements and 2^{n1} subsets with an odd number of elements.

3. Students in B.Math (hons.) first year just cannot handle Writing of Mathematics class. So n of them proceed to the football field, with chalk in hand, and stand there such that their mutual distances are all distinct. Then each of them throws the chalk at their nearest neighbour. Show that if n is odd then there is one person in the group who does not get hit by a chalk.

4. How many regions are created by n lines in the plane such that no two lines are parallel and no three lines intersect at the same point?

5. The sum of the interior angles in a triangle is 180 degrees, or π. Using this result and induction, prove that for any $n \geq 3$, the sum of the interior angles in an n-sided polygon is $(n-2)\pi$.

6. Using the method of strong induction prove that

   (a) Let be a sequence of real numbers such that $x_1 = 1, x_2 = 1$ and $x_n = x_{n-1} + x_{n-2}$ for all $n \geq 3$. Show that

   $$\sum_{k=1}^{n} x_k = x_{n+2} - 1$$

   for all $n \geq 1$.

   (b) Let be a sequence of real numbers $y_1 = y_2 = y_3 = 1$ and

   $$y_n = y_{n-1} + y_{n-2} + y_{n-3}$$

   for $n \geq 4$. Prove that $y_n < 2^n$ for all $n \geq 1$.

7. You walk into Post office at R.V. College. Ms Samiha, Chief of Post, tells you that they have only Rs 3 and Rs 7 stamp. She also informs you that your envelope will need Rs 13 worth of stamps to make it to Indian Statistical Institute in Kolkata. Determine the combination (of Rs 3 and Rs 7) that you will purchase to get stamps worth Rs 13 ? Can you determine the set of all $n \geq 1$ such that Rs n worth of stamps can be created using stamps of Rs 3 and Rs 7.

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[1]**Office hours:** I will be in my office from 9am-10am Monday, 8am-9am Tue and Thu, 10:00-11:00am Tue to answer any questions that you may have. Please feel free to drop by during these times to clarify any doubts that you may have.