Due Date: September 26th, 2019
Problems Due: 1,3

1. Let $\mathbb{R}$ be set of real numbers. Let $f : \mathbb{R} \to \mathbb{R}$. Decide which of the following statements are true or false.
   (a) If f is increasing (i.e. $f(x) < f(y)$ whenever $x, y \in R$ and $x < y$) then f is injective.
   (b) If f is increasing then f has an inverse.
   (c) If f is surjective then f is unbounded.
   (d) If f is unbounded then f is surjective.

2. $f : (0, 1) \to \mathbb{R}$ be given by
$$f(x) = \begin{cases} \frac{2x-1}{2x} & \text{if } x \leq \frac{1}{2}. \\ \frac{2x-1}{2-2x} & \text{if } x \geq \frac{1}{2}. \end{cases}$$
   Decide if f is a bijection.

3. Let $\mathbb{N}$ be the set of natural numbers.
   (a) Show that $n < 2^n$.
   (b) There is an $a \in \mathbb{N}$ and $b \in \mathbb{N}$ such that $n = 2^{a-1}(2b - 1)$
   (c) Show that Card($\mathbb{N}$) = Card($\mathbb{N} \times \mathbb{N}$)

4. Decide if Card (A) < Card (B) , Card (A) > Card (B), or Card (A) = Card (B) when
   (a) $A = (0, 1)$ and $B = [0, 1]$
   (b) $A = \mathbb{N}$ and $B = \mathbb{R}$.

Extra Credit Puzzles:

1. The class teacher of B.Math (hons.) has to decide on Siva's request of holding a class at 3am every other Saturday on non-leap years. He delegates it to a student committee of 23, with a designated chairperson. Then Siva comes to class of 40 students and ask the following question. There are two ways to do this:
   (a) Select 23 people from the class and then choose a chairperson from the selected people OR
   (b) Select a chairperson from the class and then fill out the rest of the committee.

   Find the number of ways you can do (a) and (b). Can you use the problem to obtain a general combinatorial identity with n students and k person committee with one designated chair person ?
2. How many regions are created by n lines in the plane such that no two lines are parallel and no three lines intersect at the same point?

[1] **Office hours:** I will be in my office from 9am-10am Monday, 8am-9am Tue and Thu, 10:00-11:00am Tue to answer any questions that you may have. Please feel free to drop by during these times to clarify any doubts that you may have.