Homework 5

http://www.isibang.ac.in/~athreya/Teaching/wom19 Due: September 5th and Problems Due: 3.

1. The basic logical operations are: \land (and); \lor (or); \neg (negation); \Longrightarrow (implies); and \iff (equivalent). To define them please fill in the truth table below

P	Q	$P \wedge Q$	$P \lor Q$	$P \Longrightarrow Q$	$P \Longleftrightarrow Q$
Т	Т				
Т	F				
F	Т				
F	F				

- 2. Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers. Consider the following statements :
 - (a) $\{a_n\}_{n\geq 1}$ converges to 0.
 - (c) Every subsequence of $\{a_n\}_{n\geq 1}$ has a further subsequence that converges to 0.
 - (i) Show by the direct method of proof that $(a) \Longrightarrow (c)$
 - (ii) Define what is meant by a subsequence of $\{a_n\}$.
 - (ii) Write down the statement: (c) and $\neg(c)$ in logical notation.
 - (iii) Show by the method of Proof by Contradiction that $(c) \Longrightarrow (a)$
- 3. Consider the following statements:
 - (a) For every $\epsilon > 0$ there exists N > 0 such that $|a_n L| < \epsilon$ for all n > N.
 - (b) There is a C > 0 such that for every $\epsilon > 0$ there exists N > 0 such that $|a_n L| \le C\epsilon$ for all $n \ge N$.
 - (c) For every N > 0 there exists $\epsilon > 0$ such that for all n > N implies $|a_n L| < \epsilon$.
 - (d) There exists N > 0 such that for all $\epsilon > 0$ and n > N implies $|a_n L| < \epsilon$.
 - (e) For every $\epsilon > 0$ and for all $n \ge 1$, there exists N > 0 such that m > N implies $|a_m L| < \epsilon$.
 - (f) For every $\epsilon > 0$ and for all $n \ge 1$, there exists N > 0 such that N > n and $|a_N L| < \epsilon$.

Decide which of the above versions are equivalent to the definition of

$$\lim_{n \to \infty} a_n = L$$

and which are not. For those that are not equivalent to $\lim_{n\to\infty} a_n = L$ determine, in as simple a language as possible, what they really define. Find examples (if they exist) of sequences that satisfy the definition and of sequences that don't satisfy it.

Extra Credit Puzzles:

- 1. You walk into Post office at R.V. College. Ms Samiha, Chief of Post, tells you that they have only Rs 3 and Rs 7 stamp. She also informs you that your envelope will need Rs 13 worth of stamps to make it to Indian Statistical Institute in Kolkata. Determine the combination (of Rs 3 and Rs 7) that you will purchase to get stamps worth Rs 13 ? Can you determine the set of all $n \ge 1$ such that Rs n worth of stamps can be created using stamps of Rs 3 and Rs 7.
- 2. Students in B.Math (hons.) first year just cannot handle Writing of Mathematics class. So n of them proceed to the football field, with chalk in hand, and stand there such that their mutual distances are all distinct. Then each of them throws the chalk at their nearest neighbour. Show that if n is odd then there is one person in the group who does not get hit by a chalk.