Due Date: August 29th, 2019, *Problems Due: 1,2*

1. Let $f : \mathbb{R} \to \mathbb{R}$
   - f is bounded if there exists M such that $| f(x) | \leq M$ for all $x \in \mathbb{R}$.
   - f is increasing (or strictly increasing) if $f(x) < f(y)$ whenever $x < y$.
   - f is nondecreasing if $f(x) \leq f(y)$ whenever $x < y$.
   - f is decreasing (or strictly decreasing) if $f(x) > f(y)$ whenever $x < y$.
   - f is nonincreasing if $f(x) \geq f(y)$ whenever $x < y$.

   (a) Express the statement f is not bounded without using words of negation.

   (b) Express the statement f is not increasing without using words of negation.

   (c) Compare the definitions of nonincreasing and not increasing (the latter being the negation of increasing. Does one imply the other? Are there functions that satisfy one property, but not the other?

2. Let $f, g : \mathbb{R} \to \mathbb{R}$. Determine which of the following statements are true. If true then provide a proof and if false then provide a counter example.

   (a) If f, g are bounded then $f + g$ is bounded.

   (b) If $f + g$ is bounded then f and g are bounded.

   (c) If both $f + g$ and fg are bounded then f and g are bounded.

3. Decide which of the following mathematical statements are true (*with Justification*):

   (a) 10.5 is the least upper bound of S and 9 is an upper bound of S.

   (b) Let S be finite set. If $\alpha \in \mathbb{R}$ is the least upper bound of S then $\alpha \in S$.

   (c) Let S be countable set and $\alpha \in \mathbb{R}$ be the least upper bound of S. Then for any $\beta \in \mathbb{R}$ such that $\beta < \alpha$ there is $x \in S$ such that $\beta < x < \alpha$.

   (d) Let $\alpha \in \mathbb{R}$ be the least upper bound of S. Then there exists a sequence $x_n \in S$ such that $\lim_{n\to\infty} x_n = \alpha$.

   (e) Let $\alpha \in S$ be the least upper bound of S. Then $\alpha \in S$ is not an upper bound of $T := S \setminus \{\alpha\}$.