August 1st, 2019 **Writing Of Mathematics** **Homework 3**

http://www.isibang.ac.in/∼athreya/Teaching/wom19

Due Date: August 13th, 2019, *Problems Due: 1*
Due Date: August 22nd, 2019, *Problems Due: 2, 3*

Negation of statement A: a statement B whose assertion specifically denies the truth of statement A.

1. For the following statements below write down their negations in words (like the statements).

   (a) If A holds, then B holds.

   (b) A is true only if B is true.

   (c) A is true whenever B is true.

   (d) A is false only if B is false.

   (e) A is a necessary condition for B.

   (f) A is necessary and sufficient for B.

   (g) A holds if and only if B holds

   We introduce the following **Logical Notation:** $\forall$ to mean for all; $\exists$ to mean there exists; $\implies$ to mean implies; and $\iff$ to mean equivalent. Here is an example of usage of notation.

   Statement : $f(x, y) \neq 0$ whenever $x \neq 0$ and $y \neq 0$.
   Statement in logical Notation: $\forall x, y \in \mathbb{R}$, $[x \neq 0, y \neq 0 \implies f(x, y) \neq 0]$

2. As indicated above: (i)Translate the following sentences into logical notation, (ii) negate the statement written in logical notation, and (iii) translate the negated statement back into English.

   (a) For all $M \in \mathbb{R}$ there exists $x \in \mathbb{R}$ such that $\mid f(x) \mid \geq M$.

   (b) For all $M \in \mathbb{R}$ there exists $x \in \mathbb{R}$ such that for all $y > x$ we have $f(y) > M$.

   (c) For all $x \in \mathbb{R}$ there exists $y \in \mathbb{R}$ such that $f(y) > f(x)$.

   (d) For every $\epsilon > 0$ there exists $x_0 \in \mathbb{R}$ such that $\mid f(x) \mid < \epsilon$ for all $x > x_0$.

   (e) For every $\epsilon > 0$ there exists $\delta > 0$ such that $\mid f(x) - f(y) \mid < \epsilon$ whenever $\mid x - x_0 \mid < \delta$.

3. We say $\lim_{x \to 0} f(x) = 0$ if

   For every $\epsilon > 0$ there exists $\delta > 0$ such that $\mid f(x) \mid < \epsilon$ whenever $\mid x \mid < \delta$.

   Consider the following statements:

   (a) For every $\epsilon > 0$ there exists $\delta > 0$ such that for all $x \in \mathbb{R}$, $\mid x \mid < \delta$ implies $\mid f(x) \mid < \epsilon$.

   (b) For every $\delta > 0$ there exists $\epsilon > 0$ such that for all $x \in \mathbb{R}$, $\mid x \mid < \delta$ implies $\mid f(x) \mid < \epsilon$.

   (c) There exists $\delta > 0$ such that for all $\epsilon > 0$ and for all $x \in \mathbb{R}$, $\mid x \mid < \delta$ implies $\mid f(x) \mid < \epsilon$.

   (d) For every $\epsilon > 0$ and for all $x \in \mathbb{R}$, there exists $\delta > 0$ such that $\mid x \mid < \delta$ implies $\mid f(x) \mid < \epsilon$.

   Decide which of the above versions are equivalent to the definition of

   $$\lim_{x \to 0} f(x) = 0$$

   and which are not. For those that are not equivalent determine, in as simple a language as possible, what they really define. Find examples (if they exist) of functions that satisfy the definition.