Due Date: August 8th, 2019
Problems Due: 2,5

1. Let $f(x) = 3x^5 - 9x^2$ for all $x \in \mathbb{R}$. Find the
   (a) Zeros of f, f', f''.
   (b) Identify regions where: f is positive; f' is positive; and f'' is positive.
   (c) Identify: Critical points ; inflection points; regions where the graph is concave up and down.
   (d) Identify $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$
   (e) Draw a rough-sketch of graph of f
2. Let $f(x) = x^3e^{-x}$ for all $x \in \mathbb{R}$. Find the
   (a) Zeros of f, f', f''.
   (b) Identify regions where: f is positive; f' is positive; and f'' is positive.
   (c) Identify: Critical points; inflection points; regions where the graph is concave up and down.
   (d) Identify $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$
   (e) Draw a rough-sketch of graph of f
3. Let x, y, u, v be real numbers. Prove that
$$(xu + yv)^2 \leq (x^2 + y^2)(u^2 + v^2)$$
   and determine precisely when equality will hold in the above statement.
4. Let $S = \{x \in \mathbb{R} : x(x-1)(x-2)(x-3) < 0\}$. Let $T = (0, 1)$ and $U = (2, 3)$. Obtain a simple set equality relating S, T, U.
5. Let $a \in \mathbb{R}$. We shall use the convention $(-\infty, a) = \{x \in \mathbb{R} : x < a\}$. Let $n \in \mathbb{N}$ and $a_i \in \mathbb{R}$ for $1 \leq i \leq n$ with $a_i < a_{i+1}$ for $1 \leq i \leq n-1$. Express $S = \{x \in \mathbb{R} : \prod_{i=1}^{n}(x - a_i) < 0\}$ using the notation for intervals.
6. Let $S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 100\}$ and $T = \{(x, y) \in \mathbb{R}^2 : x + y \leq 14\}$. Graph the region $S \cap T$ and count the number of elements in $S \cap T \cap \mathbb{Z}^2$.

Extra Credit[1]

1. *A positive integer is palindromic if reversing the digits of its base 10 representation does not change the number. Why is every palindromic integer with an even number of digits divisible by 11 ?*
2. *Let $x \in [0, 1]$ and we push x^2 in the calculator repeatedly. Let the sequence of numbers generated be denoted by x_n. Can you identify where the sequence* tends *to as n gets* large *? What happens if we replace x^2 by a general quadratic function ?*

[1]This problem is intended as a writing skill challenge. If you submit a well-written solution then it shall be posted on the course website.