Homework 10
Due Date: October 25th, 2019

1. Let $n \geq i$, and $\phi : \mathbb{Z} \to \mathbb{Z}_n$ be given by $\phi(i) = i \mod n$. Show that
$$\phi(i+j) = \phi(i) +_n \phi(j) \text{ and } \phi(i \cdot j) = \phi(i) \cdot_n \phi(j).$$
2. Write out the cycle decomposition of each element in S_4. Find the order of each element.
3. Find all elements of $G = \mathbb{Z}_9$ which generates G as a cyclic group. Find all subgroups of G and also find a generator in each case.
4. Let p be a prime number. Let $a \in \mathbb{N}$ not divisible by p. Show that $f : \mathbb{Z}_p \to \mathbb{Z}_p$ given by $f(x) = ax$ is a bijection.
5. Let G be a set endowed with an operation $\cdot$. It is given that there is an $e \in G$ such that $g \cdot e = g$ for all $g \in G$ and for all $g \in G$ there is a $h \in G$ such that $g \cdot h = e$. Is $(G, \cdot)$ necessarily a group ?

Extra Credit Puzzles:

1. *The coins Problem.* Given several piles of 1 Rupee coins from the ISI-Canteen cashier, we create a new collection by removing one coin from each old pile to make a new pile. Each original pile shrinks by one. That is, for example: if the original collection had four piles of 1,1,2,5 then the new set of piles will have 1,4,4.
   (a) Which lists of sizes (order is not important) are unchanged under this operation ?
   (b) Let S_n be the set of non decreasing lists summing to n. Let $f : S_n \to S_n$ be the function defined on S_n by the operation above.
      (i) Draw the functional digraph of f when n = 6.
      (ii) Determine all values of n such that f is injective. Determine all values of n such that f is surjective.
2. *The darts Problem.* Suppose a dart board has regions with values a and b, where a and b are natural numbers that are relatively prime. What is the largest integer k that cannot be achieved by summing the values of thrown darts? We seek k such that $ma + nb = k$ has no solutions in nonnegativeintegers m, n, but $ma + nb = j$ does have such a solution whenever j is an integer larger than k.

[4]**Office hours:** I will be in my office from 8:15-9am on Tue,Wed, Thu and from 2-3pm on Wed to answer any questions that you may have. Please feel free to drop by during these times to clarify any doubts that you may have.