

4.

$$a_n \rightarrow a$$

$$\text{as } n \rightarrow \infty$$

then

$$\frac{\sum_{k=1}^n a_k}{n} \longrightarrow a$$

Proof:- let $\varepsilon > 0$ be given.

$$a_n \rightarrow a \text{ as } n \rightarrow \infty \Rightarrow \exists N > 1 \text{ st } \forall n > N, |a_n - a| < \varepsilon$$

Consider $\left| \frac{\sum_{k=1}^n a_k}{n} - a \right|$

$$= \left| \frac{\sum_{k=1}^n (a_k - a)}{n} \right|$$

Induction
 (2) $\leftarrow$
 D'le
 inequality

$$\leq \frac{\sum_{k=1}^n |a_k - a|}{n}$$

$(n \geq N)$

$$= \frac{\sum_{k=1}^N |a_k - a|}{n} + \frac{\sum_{k=N+1}^n |a_k - a|}{n}$$

let $C = \sum_{k=1}^N |a_k - a|$

$$\left| \frac{a_1 + \dots + a_n}{n} - a \right| \leq \frac{C}{n} + \frac{(n-N)\varepsilon}{n}$$

$$= \frac{C}{n} + (1 - \frac{N}{n})\epsilon \quad \text{--- } \textcircled{x}$$

$$N_1 = \left\lceil \frac{C}{\epsilon} \right\rceil + 1 \quad n \geq N_1 \quad \Rightarrow \quad \frac{C}{n} < \epsilon \quad \text{--- } \textcircled{x \cdot x}$$

$$1 - \frac{N}{n} < 1 \quad \text{--- } \textcircled{x x x}$$

$$\text{take } n \geq \max(N_1, N) + 1 \quad \textcircled{x} \textcircled{x x}, \textcircled{x x x}$$

$$\Rightarrow \left| \frac{a_1 + \dots + a_n}{n} - a \right| < 2\epsilon$$

As $\epsilon > 0$ arbitrary we have
 shown $\frac{a_1 + \dots + a_n}{n} \rightarrow a$ as $n \rightarrow \infty$, p