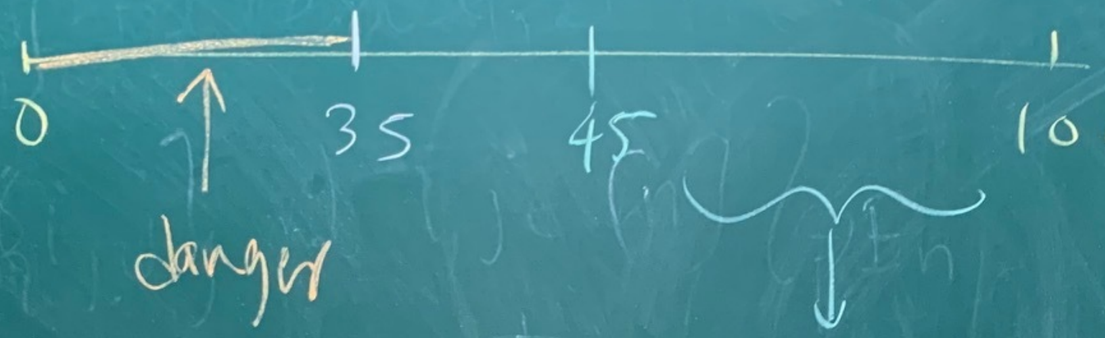

$X \doteq$ Attendance $> 85\%$

[Banana Muffins]: $\text{Card}(x \mid \text{Midterm} > 55) =$

[Bake Carrot Cake]: $\text{Card}(x \mid \text{final score} > 45) =$ $\text{Card}(x)$

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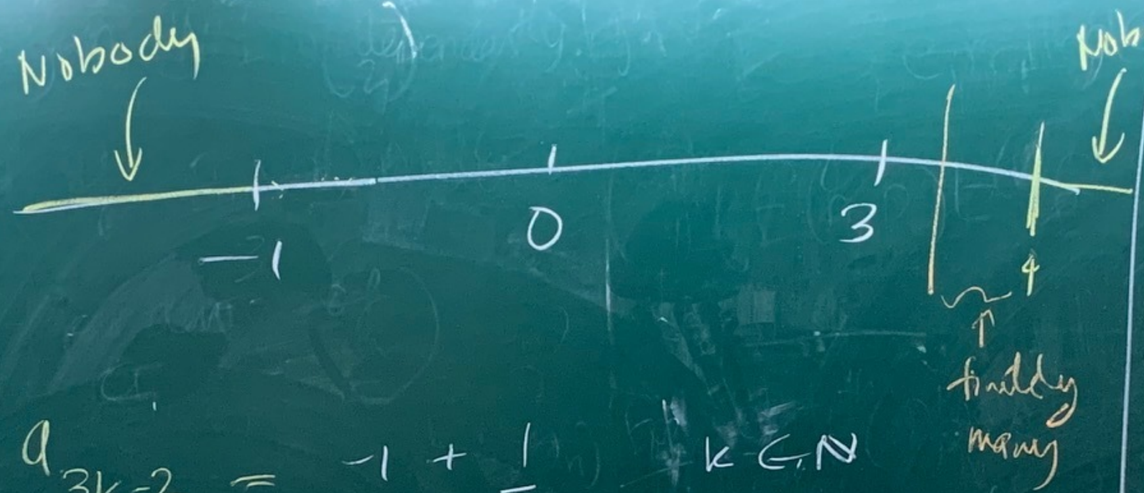

Example: $a_{3k-2} = -1 + \frac{1}{k}$ $k \in \mathbb{N}$

$a_{3k-1} = \frac{1}{k}$ $k \in \mathbb{N}$

$a_{3k} = 3 + \frac{1}{k}$ $k \in \mathbb{N}$

let $\varepsilon > 0$ be given

let $N \geq 1$ be given

$k_0 = \lceil \frac{1}{\varepsilon} \rceil + 1$

$k_1 = N + 1$

$k = \max\{k_0, k_1\}$

$n_1 = 3k-2$

$n_2 = 3k-1$

$n_3 = 3k$

$k \geq 1 \Rightarrow n_1, n_2, n_3 \in \mathbb{N}$

$(\because N \geq 1)$

and $n_1 \geq N$

$(n_1 \geq k, n_2 \geq k)$ $n_3 \geq N$ $n_2 \geq N$

$a_{n_1} = -1 + \frac{1}{k} < -1 + \varepsilon$ and $a_{n_1} > -1$ $\left(\because \frac{1}{k} > 0 \text{ and } \frac{1}{k} < \varepsilon \right)$

$a_{n_2} = \frac{1}{k} < \varepsilon$ and $a_{n_2} > 0$

$a_{n_3} = 3 + \frac{1}{k} < 3 + \varepsilon$ and $a_{n_3} > 3$

$\Rightarrow$ $a_{n_1} \in (-1-\varepsilon, -1+\varepsilon)$

$a_{n_2} \in (-\varepsilon, \varepsilon)$

$a_{n_3} \in (3-\varepsilon, 3+\varepsilon)$

As $N \geq 1$ was arbitrary & $\varepsilon > 0$ was arbitrary there are infinitely many elements of the sequence in $(-1-\varepsilon, -1+\varepsilon)$, $(-\varepsilon, \varepsilon)$, $(3-\varepsilon, 3+\varepsilon)$

(3) $f: \mathbb{R} \to \mathbb{R}$

↗ f - unbounded

$\Rightarrow \exists \{a_n\}_{n \geq 1}$ $a_n \in \mathbb{R}$, $|f(a_n)| \to \infty$ as $n \to \infty$

↘ $\forall M > 0 \quad \exists x \in \mathbb{R}, |f(x)| > M$ - (*)

let $M = 1$ $\exists x_1 \in \mathbb{R}$ s.t.

[Use (*)] $|f(x_1)| > 1$

Suppose $M \leq k$ $\exists x_1, \ldots x_k$ such that

$M \in \mathbb{N}$ $|f(x_k)| > k$

Let $R>0$ be given.

Take $N = \lfloor R \rfloor + 1$

$n \geq N \Rightarrow |f(x_n)| > n \geq N > R$ (**)

As $R>0$ was arbitrary we have shown that $|f(x_n)| \to \infty$ as $n \to \infty$. □

Take

$M = k+1$ $\exists$ $x_{k+1} \in \mathbb{R}$ s.t $|f(x_{k+1})| > k+1$

[Use ⊗]

Hence by induction $\forall n \geq 1$ $\exists x_n \in \mathbb{R}$

such that $|f(x_n)| > n$ — (**)

Show:- $|f(x_n)| \to \infty$ as $n \to \infty$

Proof:- $\forall R > 0$ $\exists N \geq 1$ such that

$\forall n \geq N$ $|f(x_n)| > R$

To show

$\frac{1}{n} \to 0$ as $n \to \infty$ ✓

$\frac{1}{2^n} \to 0$ as $n \to \infty$ $\boxed{2^n > n \text{ [induction]}}$

$\frac{n}{2^n}$ $b > 1$ $1+\delta$ for some $\delta > 0$.

$b > 1$

$\boxed{\begin{array}{l} n > 3 \;\; ? \; c > 0 \\ \frac{n^\beta}{b^n} < \frac{c}{n^2} \end{array}}$

$\boxed{b^n = (1+\delta)^n = \sum_{k=0}^{n} {}_nC_k \, \delta^k > {}_nC_2 \, \delta^2 \quad n > 3}$

${}_nC_{\lfloor \beta \rfloor + 1} \, \delta^{\lfloor \beta \rfloor + 1}$