1. Let $f(x) = \frac{x}{1+x^2}$ for all $x \in \mathbb{R}$. Find the
   (a) Zeros of f, f', f''.
   (b) Identify regions where: f is positive; f' is positive; and f'' is positive.
   (c) Identify: Critical points ; inflection points; regions where the graph is concave up and down.
   (d) Identify $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$
   (e) Draw a rough-sketch of graph of f

2. We are given $y = ax^2 + bx + c$. In each of the cases below, draw a rough sketch of y:

   (i) $b^2 - 4ac > 0$ and $a > 0$, (ii) $b^2 - 4ac > 0$ and $a < 0$,

   (iii) $b^2 - 4ac = 0$ and $a < 0$, (iv) $b^2 - 4ac = 0$ and $a > 0$,

   (v) $b^2 - 4ac < 0$ and $a < 0$, (vi) $b^2 - 4ac < 0$ and $a > 0$.
   Distinguish each w.r.t. to y attaining its global maximum or minimum and being concave up or down.

3. Let $f : \mathbb{R} \to \mathbb{R}$. Suppose f is differentiable two times, then show that

$$f(x) = f(x_0) + (x - x_0)f'(x_0) + \frac{(x - x_0)^2}{2} f''(\xi),$$

   for any $x, x_0 \in \mathbb{R}$ and ξ is a point between x and x_0. In addition, if the second derivative of f is continuous, $f'(x_0) = 0$, $f''(x_0) < 0$, then show that f has a local maximum at x_0.