

1. Define what is meant by “a sequence  $\{a_n\}_{n \geq 1}$ ”.
2. We say  $\lim_{n \rightarrow \infty} a_n = 3$  if

*For every  $\epsilon > 0$  there exists  $N > 0$  such that  $|a_n - 3| < \epsilon$  whenever  $n \geq N$ .*

- (a) Provide an example of sequence that converges to 3.
  - (b) Write a logical statement that is equivalent to saying  $\lim_{n \rightarrow \infty} a_n \neq 3$
  - (c) Provide an example of sequence that does not converges to 3.
  - (d) Provide an example of sequence that does not converges to any real number.
  - (e) Write a logical statement that is equivalent to saying that the sequence  $a_n$  does not converge to any real number.
3. A sequence  $\{a_n\}$  is a bounded sequence if there is a  $M > 0$  such that  $a_n$  is in the interval  $(-M, M)$  for all  $n \in \mathbb{N}$ .
    - (a) Write a logical statement that is equivalent to saying that the sequence  $a_n$  is bounded.
    - (b) Provide an example of a bounded sequence: which converges and which does not converge.
    - (c) Write a logical statement that is equivalent to saying that the sequence  $a_n$  is not bounded.
    - (d) Write a logical statement that is equivalent to saying that the sequence  $a_n$  diverges to  $\infty$ .
    - (e) Write a logical statement that is equivalent to saying that the sequence  $a_n$  diverges to  $-\infty$ .
    - (f) Provide an example of a sequence : that is not bounded and diverges to  $+\infty$ ; that is not bounded diverges to  $-\infty$ ; and that is not bounded and neither diverges to  $\pm\infty$ .
    - (g) Provide an example (if any) of an unbounded sequence that converges to 0.