1. Let us define the following terms:-
   - $S \subseteq R$ is bounded if there exists a positive real number $M$ such that $\mid x \mid \leq M$ for all $x \in S$.
   - $f : \mathbb{R} \to \mathbb{R}$ is bounded if there exists $M$ such that $\mid f(x) \mid \leq M$ for all $x \in \mathbb{R}$.

   Provide an example of a set $S \subseteq \mathbb{R}$ that is bounded; a set $S \subseteq \mathbb{R}$ that is not bounded; $f : \mathbb{R} \to \mathbb{R}$ that is bounded; and $f : \mathbb{R} \to \mathbb{R}$ that is not bounded.

   Write the above definitions in logical notation.

2. Write a statement in logical notation that is equivalent to saying
   (a) $f : \mathbb{R} \to \mathbb{R}$ is not bounded.
   (b) $S \subseteq \mathbb{R}$ is not bounded.

3. Decide if there exists an example of a function that satisfies (b) but not (a) and if there exists an example of a function that satisfies (a) but not (b).
   (a) For all $M \in \mathbb{R}$ there exists $x \in \mathbb{R}$ such that $\mid f(x) \mid \geq M$.
   (b) For all $M \in \mathbb{R}$ there exists $x \in \mathbb{R}$ such that for all $y > x$ we have $\mid f(y) \mid > M$.

4. Let $A \subset \mathbb{R}$.
   (a) What is meant by saying $\alpha = \sup(A)$ ?
      i. Write down a logical statement that states that $s$ is an upper bound of $A$ but not the supremum of $A$. Can such an $s$ as above be an element of $A$ ?
   (b) Find the infimum and supremum of the sets
      i. $B = \{2, 3, 4\}$
      ii. $S = \{\frac{1}{n} : n \in \mathbb{N}\} \cup \{50 + \frac{1}{n} : n \in \mathbb{N}\}$
      iii. $A = \{1 - \frac{(-1)^n}{n} : n \in \mathbb{N}\}$
   (c) Let $\alpha$ be $\sup(A)$ and $B \subset \mathbb{R}$ $\beta = \sup(B)$. Let $C = \{a \cdot b : a \in A, b \in B\}$. Is the $\sup(C) = \alpha\beta$?