Language and (Writing) Proofs

Two Broad (ideas) of Proof

Result :-
(Usually has this format)

- Hypothesis or Assumptions
- Conclusions

. Proof can be one of two ways,

Di

. Proo

– C

Direct Proof:

- Consider assumptions as given

through logical deduction

- Arrive at conclusion

Proof by Contradiction

- Consider Assumption as given AND Conclusions are False --> through logical deduction Arrive at a contradiction.

Result 1: - let $a, b, c, d, r, s \in \mathbb{R}$

let $x, y \in \mathbb{R}$ such that

$$\left.\begin{aligned} ax + by &= r \quad (1) \\ cx + dy &= s \quad (2) \end{aligned}\right\} - (*)$$

- satisfy the two linear equati

Assume $ad - bc \neq 0$ then $(*)$ has a unique solution.

Assume $ad - bc = 0$, then depending on r, s there may be ~~infinitely many solution~~ or no solution to $(*)$

$\boxed{\text{Ex: DIRECT PROOF}}$

Result 2 :- Let a, b, c be odd integers. Then

$$ax^2 + bx + c = 0 \quad - (4)$$

has no solution in the set of rational numbers.

Ex: Proof by contradiction

Key steps - (1)

(I) $a = b = c = d = 0$

$r = s \neq 0$

$r \neq s$

$r = s = 0$

(II) NLOG $d \neq 0 \rightarrow y = \dfrac{s - cx}{d} \quad - (2)$

Plug in (1) $(ad - bc)x = rd - bs$

(III) • $ad - bc \neq 0$, • $ad - bc = 0$ & $\begin{cases} rd = bs & (IV) \\ rd \neq bs & (V) \end{cases}$

Key steps - (2)

• Assume $\dfrac{p}{q}$ satisfies (4), gcd

$\rightarrow ap^2 + bpq + cq^2 = 0$

Three cases - arrive at contrad

DIRECT PROOF

- Consider assumptions as given

through logical deductions

- Arrive at conclusion

Result 1. – Let $a, b, c, d \in \mathbb{R}$

Let $x, y \in \mathbb{R}$ such that

$$\left.\begin{array}{l} ax + by = r \text{ — (i)} \\ cx + dy = s \text{ — (ii)} \end{array}\right\} \text{— (*)}$$

– satisfy the two linear equations

Assume $ad - bc \neq 0$ then (*) has a unique solution.

Assume $ad - bc = 0$, then depending on r, s there may be infinitely many solutions or no solution to (*)

$\boxed{\text{Ex. – DIRECT PROOF}}$

- **Proof by Contradiction**

– Consider Assumption as given AND Conclusions are False $\dashrightarrow$ through logical deductions → Arrive at a contradiction.

Quantifiers and logical statements

Before trying to prove any statement it is a requirement to understand [of what it means] for the statement to be false

Negation

Examples:-

(S) Everyone at the I.S.I campus is wearing a black T-shirt

What does it mean for (S) to be false?

(S') There is at least one person at the I.S.I campus who is NOT wearing a black T-shirt

ENGLISH

- Ambiguity is present [Unlike Mathematics]
- There is context
- Word order & Rules that govern statements

Rohan made Abhinav eat ISI mess lunch

Eat Abhinav ; Rohan made ISI mess lunch

Eat ISI mess lunch Rohan Abhinav made.