1.a $\quad f(x) = x^3 - 9x \quad \forall x \in \mathbb{R}$

$$f(x) = x^3 - 9x$$

$$f(x) = 0$$

$$\Rightarrow x^3 - 9x = 0$$

$$\Rightarrow x(x^2 - 9) = 0$$

$$\Rightarrow x(x-3)(x+3) = 0$$

$$\Rightarrow x = 0, 3, -3$$

$$f(x) = x^3 - 9x$$

$$f'(x) = 3x^2 - 9$$

$$f'(x) = 0$$

$$\Rightarrow 3x^2 - 9 = 0$$

$$\Rightarrow 3(x^2 - 3) = 0$$

$$\Rightarrow x^2 - 3 = 0$$

$$\Rightarrow x^2 = 3$$

$$\Rightarrow x = \pm\sqrt{3}$$

$$f'(x) = 3x^2 - 9$$

$$f''(x) = 6x$$

$$f''(x) = 0$$

$$\Rightarrow 6x = 0$$

$$\Rightarrow x = 0$$

$\Rightarrow$

f'

$(-\infty,$

f''

$(0,$

$f' > 0$

$+ + + \quad - - - \quad + + +$

$-\sqrt{3} \qquad \sqrt{3}$

$(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

$f'' > 0$

$- - - \quad + + +$

0

i.e. $(0, \infty)$

Critical Points occur when $f'(x) = 0$

We know that $f'(x) = 0$ at $x = \sqrt{3}$

$x = -\sqrt{3}$

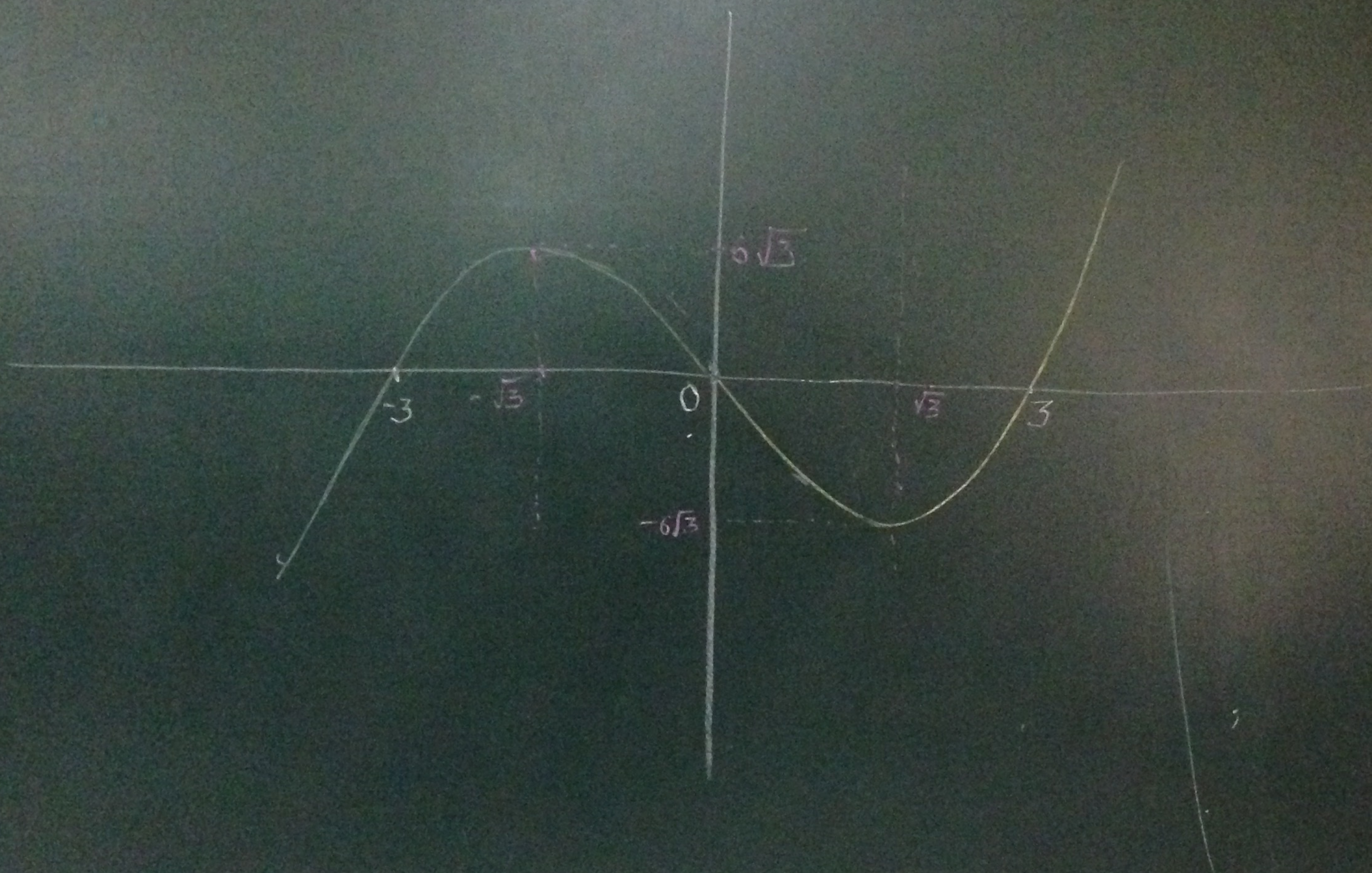
At $x = \sqrt{3}$, $f''(x) = 6(\sqrt{3}) > 0$.
Hence it is a point of local minima

At $x = -\sqrt{3}$, $f''(x) = 6(-\sqrt{3}) = -6\sqrt{3} < 0$
Hence it is a point of local maxima

Point of Inflection occurs when $f''(x) = 0$
i.e., at $x = 0$

Graph is concave up if $f''(x) = 6x > 0$
$\Rightarrow x \in (0, \infty)$

Graph is concave down if $f''(x) = 6x < 0$
$\Rightarrow x \in (-\infty, 0)$



$6\sqrt{3}$
-3
$-\sqrt{3}$
0
$\sqrt{3}$
3
$-6\sqrt{3}$