

Negate:

$\forall n \geq 1$
↓
N = natural

$\exists x \in A$
↓
universal

such that

$nx < 1$
↓
 $P(x)$

We will denote any Mathematical

Statement by P, Q, R [capital letters]

Each statement has its truth value
(mathematical)

truth values $\begin{matrix} \text{---} T & \text{--- true} \\ \text{---} F & \text{--- false} \end{matrix}$

Example

Qualifiers

$\neg P$ is used to denote negation of P .

Example [Notation]

[Real]

For all $x \in S$, $P(x)$ is true

For all $x \in \mathbb{R}$, $x^2 > 0$

Qualifiers

Universe

$P(x)$

Mathematical statement

Qualifiers are of two forms

Universal = $\begin{cases} \text{if} \\ \text{for all, let} \\ \text{for any, arbitrary} \\ \text{whenever} \end{cases}$

Existential = $\begin{cases} \text{there exists, has a} \\ \text{for some,} \\ \text{at least one,} \end{cases}$

He typically
wrote only
white
orange

ORDER

Qualifiers

helps

mathematical statement

Universal

if ...

then

must have

Existential

such that

for which

$\neg[(\forall x \in S), (P(x))]$

has
same
meaning

$(\exists x \in S), (\neg P(x))$

ORDER OF
QUALIFIERS, matter

When one negates a qualified statement, we need to understand how to pass negation through a qualifier.

Negate:

(I) $\forall n \geq 1$ $\exists x \in A$ such that $\underbrace{nx < 1}_{p(x)}$

$N \equiv \text{universe}$ universe

ORDER OF Qualifier

(I) Examples of A for which $P(x)$ is true

$A = (0, 1)$ ✓
 $A = (-5, -4)$

(I') $\exists x \in A$ such that $\forall n \geq 1$ we have $nx < 1$

A has to have a negative or 0.

Even

Th

$\neg(I)$

Negate

No

Every person in ISI is wearing blue shirt

There exists a person in ISI who is NOT wearing a blue shirt

$\neg(I)$ $\exists n \geq 1$ such that $\forall x \in A$ we have $nx \geq 1$

Negate: Every classroom has a chair that is not broken.

Notation

$R \equiv$ set of all classrooms, $C(r) \equiv$ set of all chairs in room r

$B(c) \equiv$ chair is not broken

ORDER

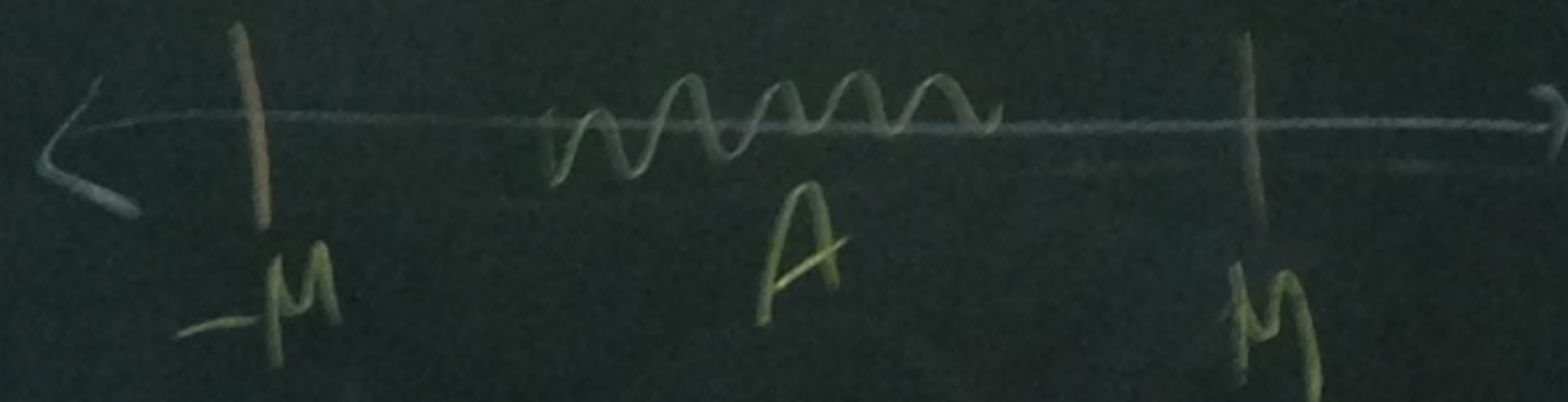
$\forall r \in R, \exists c \in C(r)$ such that $B(c)$ holds

$\exists r \in R, \forall c \in C(r), \text{ we have, } \neg (B(c) \text{ holds})$

There is a classroom in which every chair is broken.

$A \subseteq \mathbb{Z}$ is bounded

If there is $M \in \mathbb{Z}, M > 0$ such that for $a \in A$ we have $-M \leq a \leq M$



$\neg [(\forall x \in S), (P(x))]$

has
same
meaning

$(\exists x \in S), (\neg P(x))$

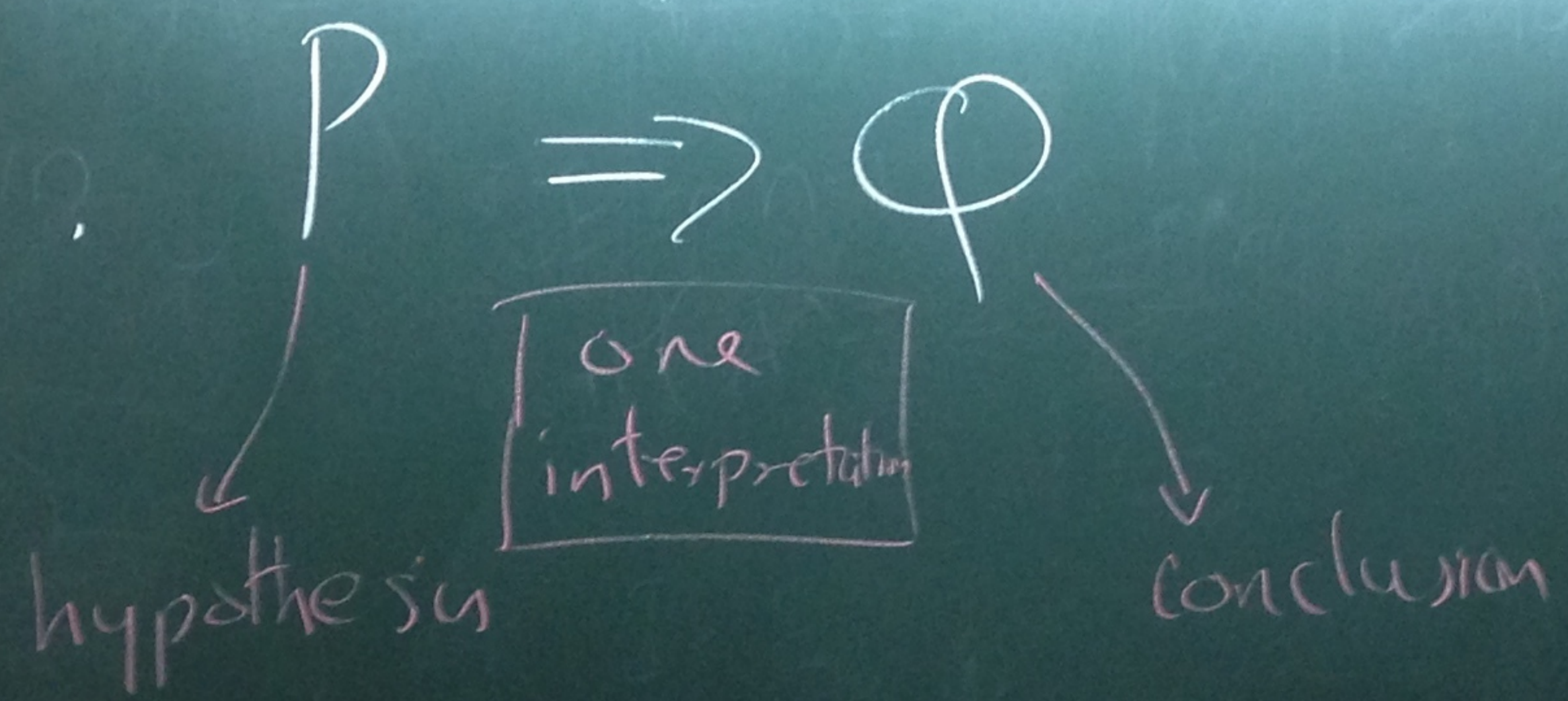
ORDER OF
QUALIFIER, matter.

Compound statements

- Logication
- and
 - or
 - implies
 - if
 - if and only if

Connectives	Notation	Meaning	Condition for Truth
Negation	$\neg P$	not P	P false
Conjunction	$P \wedge Q$	P and Q	Both are True
Disjunction	$P \vee Q$	P or Q	At least one is true
Biconditional	$P (\equiv) Q$	P if and only if Q	P, Q have same truth value
Conditional	$P \Rightarrow Q$	P implies Q	Q is true whenever P is true

Truth



Think

$Q \Rightarrow P$ is the converse
of
 $P \Rightarrow Q$

$P: x > 0$

$Q: x^2 > 0$

$R: x - 4 > -4$

$P \Rightarrow Q$ ✓

$Q \Rightarrow P$ False

Condition

$$P \Rightarrow R, R \Rightarrow P \quad (P \Leftrightarrow R)$$

$$R \Rightarrow Q.$$

Conditionals

P - Siva was Born yesterday

$P \Rightarrow Q$ (True)

Q - - - - -

$$\underline{P \Rightarrow Q}$$

If P is true then Q is true

Q is true if P is true

P is true only if Q is true

P is a Sufficient Condition for Q

Q is a Necessary Condition for P

TRUTH Table

$P \Rightarrow Q$

P	Q	$P \Rightarrow Q$	$\neg P$	$(\neg P) \vee Q$	$(P \Rightarrow Q) \Leftrightarrow ((\neg P) \vee Q)$
True	True	TRUE	False	TRUE	TRUE
True	False	False	False	False	TRUE
False	True	TRUE	True	TRUE	TRUE
False	False	TRUE	True	TRUE	TRUE