Two sample test of means (variance equal)

Let $n, m \geq 1$, $X_1, X_2, \ldots, X_n$ be i.i.d. Normal(μ_X,σ_1^2) and $Y_1, Y_2, \ldots, Y_m$ be i.i.d. Normal(μ_Y,σ_2^2).

Equal Variance: $\sigma_1 = \sigma_2$

Test Statistic:

$$T := \frac{\bar{X} - \bar{Y} - (\mu_x - \mu_Y)}{S_{pooled}\sqrt{\frac{1}{n} + \frac{1}{m}}} = \frac{\text{Observed} - \text{Expected}}{\text{Standard Error}}$$

where

$$S_{pooled}^2 = \frac{(n-1)S_X^2 + (m-1)S_Y^2}{n+m-2}$$

$T \sim t_{n+m-2}$

Two Sample-test (Welch approximation)

Let $n, m \geq 1$, $X_1, X_2, \ldots, X_n$ be i.i.d. Normal(μ_X,σ_1^2) and $Y_1, Y_2, \ldots, Y_m$ be i.i.d. Normal(μ_Y,σ_2^2).

Unequal Variance: $\sigma_1 \neq \sigma_2$

Test Statistic:

$$T := \frac{\bar{X} - \bar{Y} - (\mu_x - \mu_Y)}{S_{pooled}}$$

where s

$$S_{pooled}^2 = \frac{S_X^2}{n} + \frac{S_Y^2}{m}$$

$$T \sim t_d \text{ with } d = \left(\frac{S_X^2}{n} + \frac{S_Y^2}{m}\right)^2 \cdot \left(\frac{1}{n-1}\left(\frac{S_X^2}{n}\right)^2 + \frac{1}{m-1}\left(\frac{S_Y^2}{m}\right)^2\right)^{-1}.$$

Test of means: Paired Samples

Let $n \geq 1$, $X_1, X_2, \dots, X_n$ and $Y_1, Y_2, \dots, Y_n$ betwo samples, that are paired. Then the

Test Statistic:

$$T := \frac{\bar{X} - \bar{Y} - (\mu_x - \mu_Y)}{\frac{S}{\sqrt{n}}}$$

where

$$S^2 = \frac{1}{n-1}(\sum_{i=1}^{n}(z_i - \bar{z})^2) \qquad \text{with } z_i = x_i - y_i$$

$T \sim t_{n-1}$

We can think of it as single sample test of equality of means with data from $x_i - y_i$.

Analysis of Variance One way

I - treaments and J- population in each treatment.

Model: $y_{ij} = \mu + \alpha_i + \epsilon_{ij}$ with errors ϵ_{ij} being independent Normal $(0, \sigma^2)$ and differential effect $\sum_{i=1}^{I} \alpha_i = 0$

Null Hypothesis: $\alpha_1 = \alpha_2 = \ldots = \alpha_I = 0$

Alternative Hypothesis: One of the α_i differ from 0.

Analysis of Variance One way

I - treaments and J- population in each treatment.

$$\text{Total Sum of squares} := \sum_{i=1}^{I}\sum_{j=1}^{J}(y_{ij} - \bar{y})^2$$

$$= \sum_{i=1}^{I}\sum_{j=1}^{J}(y_{ij} - \bar{y_{i}}.)^2 + J\sum_{i=1}^{I}(\bar{y_{i}}. - \bar{y})^2$$

$:=$ Sum of squares within treatment groups

$+$ Sum of squares between treatment groups

In Short:

$$\text{SS}_{\text{total}} = \text{SS}_{\text{W}} + \text{SS}_{\text{B}}$$

Analysis of Variance One way

I - treaments and J- population in each treatment.

Test Statistic:

$$F := \frac{\mathrm{SS_B}/(I-1)}{\mathrm{SS_W}/(I(J-1)}$$

where $F \sim F(I-1, I(J-1))$

Decide on α

Calculate p-value $:= P\left(F(I-1, I(J-1)) > \frac{\mathrm{SS_B}/(I-1)}{\mathrm{SS_W}/(I(J-1)}\right)$

Reject Null Hypothesis if p-value is less that α