Elementary Hypothesis Testing

Testing :-

an observed quantity from a sample is compared to an expected result based on assumptions about the distribution

- **Null Hypothesis** :: Specific conjecture made on nature of the distribution

- **Alternative Hypothesis** :- Specifies a particular manner in which the null may be an inaccurate assumption.

Computation :- is done based on sample data and the result which would have been expected if the Null hypothesis were true.

P-value :- The above computation result in probability that sample would be at least as far from expectation as was actually observed.

- value of observation varies on computation, null & alternative

- A small p-value indicates that the sample is unusual (i.e. Null is not true)

- A large p-value indicates that

the sample is consistent.
(i.e. Null assumptions are okay).

Data: $X_1, \dots, X_n$ - say Normal σ known

Null H.

$\mu = c$

Alternate H.

$\mu > c$

Q: If null is true then how likely is it we would have seen data like $\bar{X}$?

A: let $Y_1, \dots Y_n$ i.i.d $N(c, \sigma)$

we calculate

$$P(\bar{Y} \geq \bar{X})$$

$$= P\left(\frac{\sqrt{n}(\bar{Y} - c)}{\sigma} \geq \frac{\sqrt{n}(\bar{X} - c)}{\sigma}\right)$$

$$= \mathbb{P}\left(Z \geq \frac{\sqrt{n}(\bar{X} - c)}{\sigma}\right)$$

We also decide – on a level of Significance – α

ie if $\mathbb{P}(\bar{Y} \geq \bar{X}) < \alpha$

ie $\bar{X}$ is so far from assumed mean

c that assumption $\mu = c$ is incorrect.

[– Rejecting the null hypothesis]

If $\mathbb{P}(\bar{Y} \geq \bar{X}) \geq \alpha$

i.e $\bar{X}$ is consistent with assumption

$\mu = c$ – null is not Rejected

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Example 1-

$\sigma = 3 \qquad n = 16 \qquad \bar{X} = 10.2$

Null: $\mu = 9.5 \qquad \alpha = 0.05$

(a) Alt : $\mu > 9.5$

$Y_1 \ldots Y_{16} \sim N(9.5, 3)$ [Sampled from $N(9.5, 3)$]

$$\mathbb{P}(\bar{Y} \geq \bar{X})$$

$$= \mathbb{P}\left(Z \geq \frac{4}{3}(10.2 - 9.5)\right)$$

$$\simeq 0.175$$ — called p-value

$\alpha = 0.05 \Rightarrow P(\bar{Y} \geq \bar{X}) > 0.05$

– not reject null hypothesis.

i.e.

If $\mu = 9.5$ assumption is true, the sampling procedure will produce a result at least as large as $\bar{X} = 10.2$, 17.5% of the time. This is common enough that we cannot reject the $\mu = 9.5$ assumption.

(b) Null $\mu = 8.5$ $\alpha = 0.05$

Alt $\mu > 8.5$

$$P(\bar{Y} > \bar{X}) = P\left(Z > \frac{4(10.2-8.5)}{3}\right)$$

$$\simeq 0.012$$

$\Rightarrow$ $\alpha > 0.012$

- reject the null hypothesis.

conclusion from test that $\mu > 8.5$

- ie If $\mu = 8.5$ assumption is true the sampling procedure will produce a result as large as $\bar{X} = 10.2$ only about 1.2% of the time.

This is rare enough that we can reject the hypothesis that $\mu = 8.5$.

(c) Null : $\mu = c$ σ - known

Alt : $\mu < c$

- Decide α

we would calculate

$$\mathbb{P}(\bar{Y} < \bar{X}) \left.\begin{matrix} \\ \\ \end{matrix}\right\} \begin{matrix} \geq \alpha \\ < \alpha \end{matrix}$$

and check.

Example :-

X - has a normal distribution

$n = 25$, $\bar{X} = 6.2$, $\sigma = 6$

Null : $\mu = 4$

$\alpha = 0.05$

Alt: $\mu \neq 4$

Here calculate [as no direction is specified]

$$\mathbb{P}(|\bar{Y} - 4| \geq |\bar{X} - 4|)$$

$$= 1 - P(|\bar{Y} - 4| \leq 2.2)$$

$$= 1 - P\left(|Z| < 5\frac{(2.2)}{6} \right)$$

$$\fallingdotseq 1 - P\left(|Z| < \frac{11}{6} \right) \simeq 0.0668$$

Z-test :-

σ - known $n, \bar{X}$

①	②	②
Null $\mu = c$	Null $\mu = c$	Null $\mu = c$
Alt $\mu > c$	Alt $\mu < c$	Alt $\mu \neq c$

- Decide c on 2

- Compute p-value

① - $\mathbb{P}\left(Z > \frac{\sqrt{n}(\bar{X} - c)}{\sigma} \right)$

② - $\mathbb{P}\left(Z < \frac{\sqrt{n}(\bar{X} - c)}{\sigma} \right)$

③ - $\mathbb{P}\left(|Z| < \frac{\sqrt{n}(\bar{X} - c)}{\sigma} \right)$

- Reject Null if p-value $< \alpha$.

T-test :- n; $X_1, X_2, \dots, X_n$ i.i.d from Normal population

$T - t_{n-1}$ distribution

	①	②	③
Null :	$\mu = c$	$\mu = c$	$\mu = c$
Alternate :	$\mu > c$	$\mu < c$	$\mu \neq c$

- Decide on α
- Compute p-value

① $\mathbb{P}\left(T \geq \frac{\sqrt{n}(\bar{X} - c)}{S} \right)$

② $\mathbb{P}\left(T \leq \frac{\sqrt{n}(\bar{X} - c)}{S} \right)$

③ $\mathbb{P}\left(|T| \geq \frac{\sqrt{n}(\bar{X} - c)}{S} \right)$

Reject Null if p-value $< \alpha$